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EC / EE / CS / ME / CE
General Aptitude
Quantitative Aptitude

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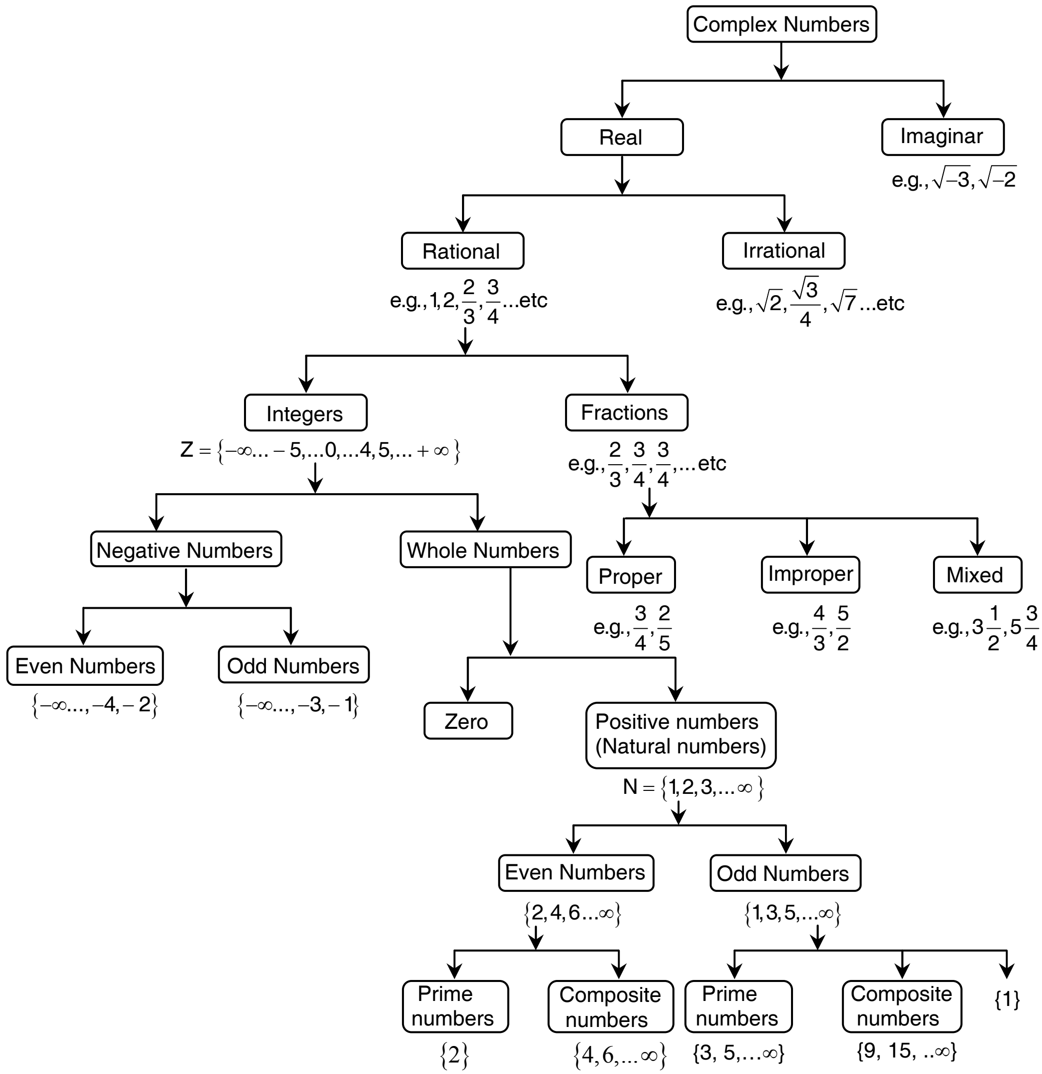
Ch.1 : Number System

Introduction

Number theory is the most elementary concept in mathematics. The scope of number theory is unlimited. It deals entirely with properties of numbers. In this book, the scope defined is limited. We have tried to bring in the concepts that have relevance to your examinations.

Number Tree:

It denotes the branching out of the numbers in the decimal system.

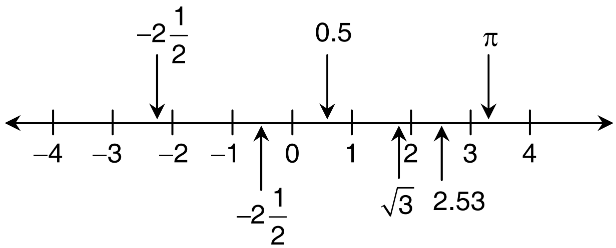


Number Tree

Complex Numbers
The general form of a complex number is **a + bi**, where a & b are real numbers and i is the imaginary unit whose value is $\sqrt{-1}$.

Imaginary Number
The general form of an imaginary number is **bi**, where b is a real number and $i = \sqrt{-1}$. In real number system, square root of a negative number does not exist which puts a constraint on the solution of certain equations, such as $x^2 + 7 = 0$. Here, $x = \sqrt{-7} = \sqrt{-1 \cdot 7} = 7i$. Hence, the imaginary numbers find a place in the number system.

Real Number
The word *number* always means real number, a number that can be represented by a point on the number line.



Number Line

The numbers to the right of 0 on the number line are called *positive* and those to the left of 0 are called *negative*.

> **Concept Key**
>
> *For any number **a**, exactly one of the following is true.*
> - *a is negative* • *a = 0* • *a is positive*

The **absolute value (modulus)** of a number a, denoted as |a|, is the distance between a and 0 on the number line. Since 3 is 3 units to the right of 0 on the number line and –3 is 3 units to the left of 0, both have an absolute value of 3.

i.e. $|3| = 3$ and $|-3| = 3$

Two unequal number that have the same absolute value are called opposites. So 3 is the opposite of –3 and –3 is the opposite of 3. The only number that is equal to its opposite is 0.

> **Concept Key**
>
> | | |
> |---|---|
> | *\| a \| = a* | *if a is positive* |
> | *\| a \| = –a* | *if a is negative* |

Rational Numbers
If a number can be expressed in the form $\frac{p}{q}, q \neq 0$, where p and q are integers, then that number can be called a rational number (‘Rational’ is derived from the word ‘ratio’.)

All integers are also rational numbers. Every terminating decimal or a repeating decimal is also a rational number, e.g. 2, 2.1, 1.3232 … etc.
Rational numbers can be further divided into two parts : integers and fractions.

Integers:
Integers are the set of all non–fractional numbers lying between $-\infty$ to $+\infty$.
Integers are denoted by Z or I.
The set Z includes the elements $\{-\infty\ldots, -3, -2, -1, 0, 1, 2, 3\ldots +\infty\}$

Consecutive Integers are two or more integers written in sequence in which each integers is 1 more than the preceding integer. They are represented as n, n + 1, n + 2, …
e.g. 5, 6, 7, 8 are four consecutive integers.

Natural Numbers :
Natural numbers are the set of all positive integers. Natural number are denoted by N.
$N = \{1, 2, 3, 4, \ldots \infty\}$

Whole Numbers:
Set of whole numbers is the set of all natural numbers and 0. Whole number are denoted by
$W = \{0, 1, 2, 3, \ldots \infty\}$

Even and Odd Numbers :
Natural numbers that are divisible by 2 are called even numbers.
e.g. 2, 4, 6, 8, …

Natural numbers that are not divisible by 2 are called odd numbers.
e.g. 1, 3, 5, …

Properties relating to even and odd numbers:
All even numbers can be expressed as $(2 \times n)$ where n is natural number or $n \in N$($\in$: belongs to).
All odd numbers can be expressed as $2n + 1$ where n is a whole number ($n \in W$) or $2n - 1$ where n is a natural number ($n \in N$).

1. Sum of two odd numbers is always even.
2. Sum of two even numbers is always even.
3. Sum of odd and even number is always odd.
4. Product of two even numbers is even.
5. Product of two odd numbers is odd.
6. Product of odd and even numbers is even.
7. The base determines whether the final result is even or odd.
   (i) Even number raised to even number is even
   (ii) Even number raised to odd number is even
   (iii) Odd number raised to odd number is odd
   (iv) Odd number raised to even number is odd

Concept Key

- *The terms odd and even apply only to integers.*
- *Every integer (positive, negative, or 0) is either odd or even.*
- *0 is an even integer; it is a multiple of 2. ($0 = 0 \times 2$)*

Prime Numbers :
The numbers that have only two distinct factors, the number itself and 1, are called prime numbers. e.g. 2, 3, 5, 7, …

Factorial:
A factorial is a number obtained by multiplying all the positive integers less than or equal to a given positive integer. The factorial of a given integer 'n' is usually written as n! or $|\underline{n}$.

$n! = 1 \times 2 \times 3 \times 4 \times .... \times n$
e.g.
$1! = 1$
$2! = 1 \times 2 = 2$
$3! = 1 \times 2 \times 3 = 6$
$4! = 1 \times 2 \times 3 \times 4 = 24$
$5! = 1 \times 2 \times 3 \times 4 \times 5 = 120$

Note:
1. By convention, (zero)! = unity i.e., 0! = 1
2. Factorial is defined only for whole numbers.
3. $n! = n \times (n - 1)!$

Fibonacci Numbers:
Fibonacci Sequence is a form of sequence $\{a_n\}$ where $a_{n+2} = a_{n+1} + a_n$.
The terms of this sequence are called as Fibonacci numbers.
e.g. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, …

Rules of cyclicity
When a number is raised to some power, the unit's place digit of the result follows a particular pattern. This pattern is given by the rule of cyclicity.
The rule of cyclicity for numbers which end with digits 0–9 can be devised by the following table:

Number 'a' ending with	a^{4n+1}	a^{4n+2}	a^{4n+3}	a^{4n}
0	0	0	0	0
1	1	1	1	1
2	2	4	8	6
3	3	9	7	1
4	4	6	4	6
5	5	5	5	5
6	6	6	6	6
7	7	9	3	1
8	8	4	2	6
9	9	1	9	1

Concept Key

- *1 is not a prime number. Though it is divisible by itself and 1, the factors are not distinct*
- *2 is the only even prime number.*

Identifying a Prime Number:
For small numbers, we could find by checking i.e. if that number is divisible by any prime number till that number itself, then that number is not a prime. Else it is a prime numbers. But for large numbers like 251, we will use the following method.

Step 1 : Find the approximate square root of the given number. i.e. $\sqrt{251} \simeq 16$
Step 2 : Check if any prime number from 2 to 16 divides 251. The prime numbers from 2 to 16 are 2, 3, 5, 7, 11 and 13. Since none of these numbers divides 251 exactly, 251 is a prime number.

Composite Numbers :
A composite number is a number, which has factors besides itself and unity.
e.g. 14, 36, 45, …

Concept Key

1. *Sum of 2 odd numbers is always even. e.g. 1 + 3 = 4.*
2. *Sum of 2 even numbers is always even. e.g. 2 + 4 = 6*
3. *Sum of odd and even number is always odd. e.g. 1 + 2 = 3*
4. *Product of 2 even numbers is even. e.g. 2 × 4 = 8*
5. *Product of 2 odd numbers is odd. e.g. 1 × 3 = 3*
6. *Product of odd and even numbers is even. e.g. 2 × 3 = 6*
   1 is neither prime nor composite.

Q. "The value of the expression $2n^6 + n^4 + n^2 + 1$ is always odd where n is an integer." State True or False with Justifications.

A. True.
$2n^6$ is always even
Consider,
$n^4 + n^2$
If n is odd then, n^4 = odd
n^2 = odd
$n^4 + n^2$ = odd + odd = even
If n is even, then definitely
$n^4 + n^2$ is even.
∴ For any integer 'n', $n^4 + n^2$ is even.
∴ The given expression becomes,

$$\underline{2n^6} + \underline{n^4 + n^2} + 1$$

even + even + 1 = odd.

Fractions:

Fraction is a part of an integer e.g. $\frac{1}{5}$ means one–fifth of the whole.

The value of a fraction is not altered by multiplying or dividing the numerator and the denominator by the same number (other than 0).

e.g. $\frac{3}{4} = \frac{3}{4} \times \frac{5}{5} = \frac{15}{20}$, $\quad \frac{15}{20} = \frac{15}{20} \div \frac{5}{5} = \frac{3}{4}$

An integer can be expressed as a fraction with any denominator.

e.g. $29 = \frac{29 \times 12}{12} = \frac{348}{12}$

Fractions are primarily of five types.

i) Proper fraction : A proper fraction is one whose numerator is less than its denominator

e.g. $\frac{3}{4}, \frac{2}{5}$

ii) Improper fraction : An improper fraction is one whose numerator is equal or greater than its denominator.

e.g. $\frac{4}{3}, \frac{5}{2}$

iii) Mixed fraction : A mixed fraction consists of two parts, an integer and a fraction.

e.g. $3\frac{1}{2}, 5\frac{3}{4}$

iv) Compound fraction : A fraction of a fraction is called a compound fraction.

e.g. $\frac{3}{5}$ of $\frac{2}{7}$

v) Complex fraction

Any complicated combination of the other type of fractions

e.g. $2\frac{1}{3}\text{ of }\frac{3}{1+\frac{2}{3}}; \frac{4}{7}\text{ of }\frac{3}{3+\frac{2}{2+\frac{1}{1+3}}}$

All mixed fractions can be converted into improper fractions

e.g. $2\frac{1}{3} = \frac{7}{3}$, $8\frac{4}{5} = \frac{44}{5}$

- All improper fraction can be converted into mixed fractions

e.g. $\frac{17}{4} = \frac{4\times4+1}{4} = \frac{16}{4}+\frac{1}{4} = 4+\frac{1}{4} = 4\frac{1}{4}$

Concept Key

Comparing fractions

1. *If the fractions have the same denominator, the fraction with the larger numerator is greater*

   e.g. $\frac{9}{10} > \frac{7}{10}$

2. *If the fractions have the same numerator, the fraction with the smaller denominator is greater.*

   e.g. $\frac{3}{5} > \frac{3}{10}$

3. *If the fractions do not satisfy the above conditions, then cross multiply for comparison.*

   e.g. $\frac{1}{3}$ ⤨ $\frac{3}{8}$ *Since* $3 \times 3 > 8 \times 1$, $\frac{3}{8} > \frac{1}{3}$.

Concept Key

To determine if two fractions are equivalent, cross multiply. The fractions are equivalent if two products are equal.

e.g. $\frac{3}{8}$ and $\frac{9}{24}$ *are equivalent since* $3 \times 24 = 9 \times 8$.

A fraction is in lowest terms if no positive integer greater than 1 is a factor of both the numerator and denominator.

e.g. $\frac{12}{19}$ is in lowest terms.

Every fractions can be reduced to lowest terms by dividing the numerator and denominator by their highest common factor (HCF).

Concept Key

Arithmetic Operation with fractions

1. *To multiply two fractions, multiply their numerators and multiply their denominators.*

   e.g. $\frac{3}{5} \times \frac{6}{7} = \frac{3 \times 6}{5 \times 7} = \frac{18}{35}$

2. *To divide any number by a fraction, multiply that number by the reciprocal of the fraction.*

   e.g. $10 \div \frac{2}{3} = 10 \times \frac{3}{2} = 15$

3. *To add or subtract fractions with the same denominator, add or subtract the numerators and keep the denominator as it is.*

   e.g. $\frac{7}{3} + \frac{4}{3} = \frac{11}{3}$ *and* $\frac{7}{3} - \frac{2}{3} = \frac{5}{3}$

4. *To add or subtract fractions with different denominators, first rewrite the fraction as equivalent fractions with the same denominators.*

   e.g. $\frac{1}{6} + \frac{3}{4} = \frac{2}{12} + \frac{9}{12} = \frac{11}{12}$

Concept Key

Arithmetic operation with mixed fractions

1. *To multiply or divide mixed fractions, change them to improper fractions*

   e.g. $5\frac{1}{2}\times 3\frac{1}{4}=\frac{11}{2}\times\frac{13}{4}=\frac{143}{8}=17\frac{7}{8}$

Note :*Be aware that* $3\times 5\frac{1}{2}$ is not $15\frac{1}{2}$; *rather :*

$3\times 5\frac{1}{2}=3\left(5+\frac{1}{2}\right)=15+\frac{3}{2}=15+1\frac{1}{2}=16\frac{1}{2}$.

2. *To add mixed fractions, add the integers and add the pure fractions separately.*

   e.g. $5\frac{1}{2}+3\frac{1}{4}=(5+3)+\left(\frac{1}{2}+\frac{1}{4}\right)=8+\frac{3}{4}=8\frac{3}{4}$

3. *To subtract mixed fractions, subtract the integers and the pure fractions separately. However, if the fractional part of the second number is greater than the first number borrow 1 from the integer part.*

   e.g. since $\frac{2}{3}>\frac{1}{4}$ *we can't subtract* $5\frac{1}{4}-3\frac{2}{3}$ *until we borrow from the 5 :*

   Now, you have

   $5\frac{1}{4}-3\frac{2}{3}=4\frac{5}{4}-3\frac{2}{3}=(4-3)+\left(\frac{5}{4}-\frac{2}{3}\right)=1+\left(\frac{15}{12}-\frac{8}{12}\right)=1\frac{7}{12}$.

Concept Key

If $\frac{a}{b}$ *is the fraction of the whole that satisfies some property, then* $1-\frac{a}{b}$ *is the fraction of the whole that does not satisfy it.*

Q. In a class $\frac{1}{5}$ of the students are Americans, $\frac{1}{3}$ of the students are Russians and $\frac{1}{4}$ of the students are Indians. What fraction of the class are neither Americans, Russians nor Indians?

A. The American, Russian and Indian students constitute

$\frac{1}{5}+\frac{1}{3}+\frac{1}{4}=\frac{12}{60}+\frac{20}{60}+\frac{15}{60}=\frac{47}{60}$

of the total, so

$1-\frac{47}{60}=\frac{13}{60}$ of the class (total students) are neither Americans, Russians nor Indians.

Q. What is the value of $\dfrac{2}{2+\dfrac{2}{2+\dfrac{2}{2+...\infty}}}$

A. Let the value of given expression be x.

Then, $x = \dfrac{2}{2+x}$

$\therefore x^2 + 2x - 2 = 0$

Solving the quadratic equation, we get

$x = \left(-1+\sqrt{3}\right)$ or $\left(-1,-\sqrt{3}\right)$. But,

since x > 0, the value of $x = -1 + \sqrt{3}$

Irrational Numbers

An Irrational numbers are numbers that cannot be expressed in the form $\frac{p}{q}$ where p & q are integers and $q \neq 0$. In other words, for any real number if the decimal part is non–recurring and non–terminating then that number is called an Irrational number.

e.g. $\sqrt{2} \approx 1.414....$ (non–recurring and non–terminating), $\sqrt[3]{5}$.

Concept Key

- *All square roots of non perfect squares e.g.* $\sqrt{3}$ *, all cube roots of non perfect cubes and so on are all irrational numbers.*
- *π = 3.14159265…is irrational ($\frac{22}{7}$ is a rational number but π which is approximated as $\frac{22}{7}$ is irrational)*

Rationalization:

Consider a fraction of type $\frac{1}{\sqrt{2}}$ or $\frac{1}{\sqrt{7}+\sqrt{5}}$. Here the denominator is an irrational number.

Its very difficult to perform any mathematical operation using such numbers. Hence we convert the denominator of this number to a rational number. This is done by multiplying the denominator by its conjugate. This process is called as **rationalization.**

e.g.

1. $\dfrac{1}{\sqrt{2}} = \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

2. $\dfrac{1}{\sqrt{7}+\sqrt{5}} = \dfrac{1}{\sqrt{7}+\sqrt{5}} \times \dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \dfrac{\sqrt{7}-\sqrt{5}}{2}$

3. $\dfrac{1}{3-\sqrt{2}} = \dfrac{1}{3-\sqrt{2}} \times \dfrac{3+\sqrt{2}}{3+\sqrt{2}} = \dfrac{3+\sqrt{2}}{9-2} = \dfrac{3+\sqrt{2}}{7}$

Basic Arithmetic Operations:

In arithmetic we are basically concerned with the addition, subtraction, multiplication, and division of numbers which are explained in the table given below.

Operation	Symbol	Result	Example
Addition	+	Sum	12 + 6 = 18
Subtraction	–	Difference	12 – 6 = 6
Multiplication	$\times$	Product	$12 \times 6 = 72$
Division	$\div$	Quotient	$12 \div 6 = 2$

The sum, difference, and product of two integers are always integers; the quotient of two integers may be an integer, or a fraction. The quotient $23 \div 10$ can be expressed as $\frac{23}{10}$ or $2\frac{3}{10}$ or 2.3. If the quotient has to be an integer, then here we can also say that the quotient is 2 and there is a remainder of 3. It depends upon our point of view.

Concept Key

- *The product and quotient of two positive numbers or two negative numbers are positive.*
- *The product and quotient of a positive number and a negative number are negative.*
-

$\times$	+	–
+	+	–
–	–	+

$\div$	+	–
+	+	–
–	–	+

$12 \times 6 = 72$ $\quad 12 \times (-6) = -72$ $\quad (-12) \times 6 = -72$ $\quad (-12) \times (-6) = 72$

$12 \div 6 = 2$ $\quad 12 \div (-6) = -2$ $\quad (-12) \div 6 = -2$ $\quad (-12) \div (-6) = 2$

Concept Key

To determine whether a product of more than two numbers is positive or negative, count the number of negative factors.

- *The product of even number of negative factors is positive.*
- *The product of odd number of negative factors is negative.*

VBODMAS:

When we have to perform a series of mathematical operations, there is a rule regarding the order in which we should perform these operations. This rule is VBODMAS rule.

V – Viniculum (bar)
B – Bracket
O – Of (multiplication)
D – Division
M – Multiplication
A – Addition
S – Subtraction

Q. Simplify $\frac{5+5+5\div 5}{(5+5+5)\div 5}$

A. $\frac{5+5+5\times\frac{1}{5}}{15\div 5}=\frac{5+5+1}{15\times\frac{1}{5}}=\frac{11}{3}=3\frac{2}{3}$.

Q. Find the value of $15\div\left(2\frac{1}{2}-\overline{3\frac{1}{2}-2\frac{1}{2}}\right)$

A. $15\div\left(2\frac{1}{2}-1\right)=15\div 1\frac{1}{2}=15\div\frac{3}{2}=15\times\frac{2}{3}=10$.

Q. Evaluate $\left[\frac{6}{7}+\frac{7\frac{3}{4}-6\frac{1}{2}}{2\frac{1}{3}\times 9}\right]$ of Rs. 42

A. $\left[\frac{6}{7}+\frac{\frac{31}{4}-\frac{13}{2}}{\frac{7}{3}\times 9}\right]$ of Rs. 42

$=\left[\frac{6}{7}+\frac{\frac{31-26}{4}}{21}\right]$ of Rs.42 $=\left(\frac{6}{7}+\frac{5}{4}\times\frac{1}{21}\right)$ of Rs.42

$=\left(\frac{6}{7}+\frac{5}{84}\right)$ of Rs.42 $=\left(\frac{72+5}{84}\right)$ of Rs.42

$=\frac{77}{84}\times$ Rs.42 = Rs. $\frac{77}{2}$ = Rs.38.50.

Test for Divisibility:

1. A number is divisible by 2, when its unit's digit is 0, 2, 4, 6 or 8. e.g., 244, 350, 7916, etc.

2. number is divisible by 3,when the sum of its digits is divisible by 3. e.g., 342, 13791.

3. A number is divisible by 4 when the number formed by the last two right hand digits is divisible by 4, or if the last two digits are 0. e.g., 1264, 1500.

4. A number is divisible by 5, when its unit's digit is 5 or 0. e.g., 245 or 50.

5. A number is divisible by 6, when it is divisible by 2 and 3 both. e.g., 354.

6. A number is divisible by 7, if the number of tens added to five times the number of units is divisible by 7.
e.g., 343. Number of tens 34, 5 times number of units = $5\times 3=15$.
34 + 15 = 49 is divisible by 7.
$\therefore$ 343 is divisible by 7. In fact, $7^3=343$

7. A number is divisible by 8 when the number formed by the last three right hand digits is divisible by 8, or when the last three digits are 0's. e.g., 1000, 67895432.

8. A number is divisible by 9, when the sum of its digits is divisibly by 9, e.g., 39537.

9. A number is divisible by 10, when its unit's digit is 0. e.g., 520, 1350.

10. A number is divisible by 11, when the difference between the sum of the digits in the odd places and the sum of the digits in the even places is 0 or a multiple of 11.
e.g., 9372836.
Here $(9 + 7 + 8 + 6) = 30$ and $(3 + 2 + 3) = 8$.
$30 - 8 = 22$ which is multiple of 11.

Note:
When any number with even number of digits is added to its reverse, the sum is always a multiple of 11.

11. A number is divisible by 12, when it is divisible by 3 and 4 both. e.g., 624.

12. A number is divisible by 13, if the number of tens added to four times the number of units is divisible by 13.
e.g., 1235 Number of tens = 123.
Four times number of units = $4 \times 5 = 20$.
$123 + 20 = 143$ which is divisible by 13.
$\therefore$ 1235 is also divisible by 13.

13. A number is divisible by 14, if it is divisible by 2 as well as 7.

14. A number is divisible by 15, when it is divisible by 3 and 5 both e.g., 930.

15. A number is divisible by 16, if the number formed by last 4 digits is divisible by 16.

16. A number is divisible by 17, if the number of tens added to twelve times the number of units is divisible by 17.
e.g., 1105 Number of tens = 110;
Twelve times the number of units = $12 \times 5 = 60$
$\therefore 110 + 60 = 170$ which is divisible by 17. $\therefore$ 1105 is divisible by 17.

17. A number is divisible by 19, if the number of tens added to twice the number of units is divisible by 19. e.g., 475.
Number of tens = 47.
Twice the number of units = $2 \times 5 = 10$.
$47 + 10 = 57$ which is divisible by 19. $\therefore$ 475 is also divisible by 19.

18. A number is divisible by 24, if it is divisible by 3 and 8.

19. A number is divisible by 25, when the number formed by the last two right hand digits is divisible by 25, e.g., 123475.

20. A number is divisible by 29, if the number of tens added to thrice the number of units is divisible by 29.
e.g., 1537. Number of tens = 153
Thrice the number of units = $3 \times 7 = 21$
$153 + 21 = 174$

In 174, number of tens = 17
Thrice number of units = $3 \times 4 = 12$
17 + 12 = 29 which is divisible by 29.
$\therefore$ 174 is divisible by 29 and hence 1537 is divisible by 29.

Q. Find (a + b), if 3478a9b is divisible by 3 and 11 both.

A. Sum of digits = 3 + 4 + 7 + 8 + a + 9 + b = 31 + a + b is divisibly by 3.
$\therefore$ a + b could be 2, 5, 8, 11, 14 or 17.
Also (3 + 7 + a + b) – (4 + 8 + 9) = 0 or multiple of 11
$\therefore$ (10 + a + b) – (21) = a + b – 11 = could be 0 or multiple of 11
$\therefore$ a + b = 11.

Q. A five digit number is made using digits 2, 8, 4, 7 and 6 not necessarily in that order. Is that number divisible by 9?

A. The sum of digits of the five digit number = 2 + 8 + 4 + 7 + 6 = 27, which is divisible by 9. Hence the number is divisible by 9.
Note : The order of digits is not necessary to calculate the answer.

Q. What least number must be added to 11943 to get a number exactly divisible by 29.

A.

```
        411
    29 | 11943
        − 116
         0034
        −  29
           53
        −  29
           24 ← Remainder
```

On dividing 11943 by 29, the remainder is 24.
$\therefore$ The number to be added = (29 – 24) = 5

Rules to remember

A number n divisible by a number m can be spilt into two parts both divisible by m or two parts both non–divisible by m. No number n divisible by a number m can be spilt into two parts such that if one is divisible by m the other is non–divisible by m.
e.g.
a + b such that if a is divisible by m, b is divisible by m
p –q such that if p is divisible by m, q is divisible by m
r + s such that if r is non–divisible by m, s is non–divisible by m
x – y such that if x is non–divisible by m, y is non–divisible by m

Additional Rules on Numbers

1. If n is an odd number then $n (n^2 – 1)$ is divisible by 24.
2. If n is an odd prime number greater than 3 then $(n^2 – 1)$ is divisible by 24.
3. If n is an odd number, then $(2^n + 1)$ is divisible by 3.
4. If n is an even number, then $(2^n – 1)$ is divisible by 3.
5. If n is an odd number, then $(2^{2n} + 1)$ is divisible by 5.
6. If n is an even number, then $(2^{2n} –1)$ is divisible by 5.

7. If n is an even number, then $(2^{2n} - 1)$is divisible by 15.
8. If n is an odd number, then $(5^{2n} + 1)$ is divisible by 13.
9. If n is an even number, then $(5^{2n} - 1)$ is divisible by 13.
10. If n is any natural number, then $(5^{2n} - 1)$ is divisible by 24.
11. If n is co–prime to 5, then $n(n^4 - 1)$ is divisible by 30.
12. $(x^n + y^n)$ is divisible by $(x + y)$ when n is an odd number.
13. $(x^n - y^n)$ is divisible by $(x + y)$ when n is an even number.
14. $(x^n - y^n)$ is divisible by $(x - y)$ when n is an odd or an even number.
15. i) If p is a prime number then for any whole number a, $a^p - a$ is divisible by p.
    ii) 1 is not a prime number.

L.C.M. and H.C.F.:

Factors and Multiples

An integer n is called a factor or a divisor of another integer m if n divides m exactly (i.e when n divides m the reminder is zero). And we say that m is a multiple n.

e.g.
- i) 24 is the factor of 144
- ii) 5 and 9 are factors of 90
- iii) 51 is the multiple of 17

To understand what multiples are, let's just take an example of multiples of 3.
The multiples are 3, 6, 9, 12, … so on. We find that every successive multiple appears as the third number after the previous.

Finding the number of multiples

If one wishes to find the number of multiples of 6 less than 255, we could arrive at the number through $\frac{255}{6} = 42$ (and the remainder 3).

The remainder is of no consequence to us. So in all there are 42 multiples.

If one wishes to find the multiples of 36, find $\frac{255}{36} = 7$ (and the remainder 3).

Hence, there are 7 multiples of 36.

Factorization

It is the process of splitting any number into the form where it is express as the product of most basic prime factors.

For example, $12 = 2^2 \times 3^1$. 12 is here expressed in the factorised form in terms of its basic prime factors. This is the factorised form of 12.

It is possible to find the number of factors of a composite number without listing all those factors, from its factorised form.

Take 12 for instance, it can be expressed as $12 = 2^2 \times 3^1$.

The factors of 12 are : $\left(2^0 \times 3^1\right), \left(2^0 \times 3^0\right), \left(2^1 \times 3^1\right), \left(2^1 \times 3^0\right), \left(2^2 \times 3^1\right), \left(2^2 \times 3^0\right)$

Here the powers of 2 can be one of (0, 1, 2) and the powers of 3 can be one of (0, 1). So the total possibilities if you take the two as combination is $3 \times 2 = 6$. Each combination of the powers of 2 and 3 gives a distinctly different factor. Hence, since there are 6 different combinations of the powers of 2 and 3, there are 6 distinct factors of 12.

Concept Key

For any natural number N which can be expressed as $N = a^m \times b^n \times c^p$ where a, b, c are primes, then total number of factors of N including 1 and N is equal to $(m+1)(n+1)(p+1)$

The number of divisors of a given number N (including 1 and the number itself) where
$N = a^m b^n c^p$ where a, b, c are prime numbers, are $(1 + m)(1 + n)(1 + p)$ and the sum of
the divisors is

$$\frac{a^{m+1}-1}{a-1} \cdot \frac{b^{n+1}-1}{b-1} \cdot \frac{c^{p+1}-1}{c-1}$$

e.g. : $100 = 2 \times 2 \times 5 \times 5 = 2^2 \times 5^2$

$\therefore$ Total number of factors of $100 = (2 + 1)(2 + 1) = 3 \times 3 = 9$

i.e., 100 has 9 factors which are 1, 2, 4, 5, 10, 20, 25, 50, 100.

Highest Common Factor (H.C.F.)

A number which is a factor of two or more numbers is said to be a common factor, or common measure, of the numbers.
The Greatest Common Factor (GCF) or Highest Common Factor (HCF) of two or more integers is the largest integer that is a factor of each of them.
For example, it is required to find the H.C.F. of 8 and 12.
Factors of 8 are 1, 2, 4, 8 and
Factors of 12 are 1, 2, 3, 4, 6, 12
The common factors are 1, 2, 4; but the highest of these is 4, hence 4 is the H.C.F.

Least Common Multiple (L.C.M.)

A number which is a multiple of two or more numbers is said to be a common multiple of the numbers.
The Least Common Multiple (LCM) of two or more integers is the smallest positive integer that is a multiple of each of them. We exclude zero, which is a common multiple of all numbers.
For example, it is required to find the LCM. of 8 and 12.
Multiple of 8 are : 8, 16, 24, 32, 40, 48, …
Multiple of 12 are : 12, 24, 36, 48, ….
The common multiples are 24, 48 … but the smallest of these is 24, hence 24 is the LCM.

Concept Key

The product of the HCF and LCM of two numbers is equal to the product of the two numbers.
i.e. Product of two numbers = (HCF) × (LCM)

For any perfect square – odd no. of factors
For any other no. – Even no. of factors
1. Least number exactly divisible by x, y, z → LCM[x, y, z]
2. Least no. when divided by x, y, z leaves a remainder 'r' in each case → LCM (x, y, z) + r.
3. Least no. when divided by x, y, z leaves remainder a, b, c
x − a = y − b = z − c = k → LCM (x, y, z) − k

Q. The product of two numbers is 600. If their LCM is 120 then find their HCF.

A. $\text{HCF} = \dfrac{\text{Product of two numbers}}{\text{LCM}} = \dfrac{600}{120} = 5$

Methods to find HCF of given numbers :

1. By method of factorization
Express each number as the product of primes. Now take the product of factors which are common to all the numbers. The product is the required HCF.

Q. Find the HCF of 108, 120, 156.
A. 108 = ② × ② × ③ × 3 × 3
120 = ② × ② × 2 × ③ × 5
156 = ② × ② × ③ × 13
HCF = 2 × 2 × 3 = 12

2. By division method (synthetic method) :
Suppose two numbers are given. Divide the greatest number by the lesser; divide the lesser by the remainder; divide the first remainder by the new remainder, and so on till there is no remainder. The last divisor is the required H.C.F. This method is used to find the HCF of two numbers only.

Q. Find the HCF of 24 and 44.
A. The synthetic method for finding the HCF of 24 and 44 is as follows :

```
24 ) 44 ( 1
     24
     20 ) 24 ( 1
          20
           4 ) 20 ( 5
               20
                0
```

Since the remainder in the last stage is 0, the divisor at this stage i.e. 4 becomes the HCF of these two numbers.

Q. Resolve 9163 into prime factors.

A. Taking the primes in order we find that the divisibility tests for 2, 3, 5 are not satisfied. We next try 7 as often as possible and then 11. The quotient 17 is prime.

7	9163
7	1309
11	187
	17

$\therefore 9163 = 7 \times 7 \times 11 \times 17 = 7^2 \times 11 \times 17$

Concept Key

If there are more than 2 numbers, say 4 numbers, find the HCF of any 2 numbers and the HCF of the other 2 numbers. The HCF of their HCFs gives the HCF of all the 4 numbers.

Q. Find the HCF of 18, 24, 36 and 48

A. The HCF of the numbers 18 and 36 is 18
The HCF of the numbers 24 and 48 is 24
The HCF of the HCFs, i.e. HCF of 18 and 24 is 6
This is the HCF of all the 4 numbers.

Methods to find LCM of given numbers:

1. By method of factorization :

Express each given number as the product of primes. Now take the product of highest powers of all factors, that occurs in these numbers. The product is the required LCM.

Q. Find the LCM of 108, 120, 156.

A. $108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$

$120 = 2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3 \times 5$

$156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$

$\text{LCM} = 2^3 \times 3^3 \times 5 \times 13 = 14040$

2. By using the formula :

If two numbers are given, find their HCF and their LCM using

$$\text{LCM} = \frac{\text{Product of two numbers}}{\text{HCF}}$$

Q. Find the LCM of 24 and 44

A. $\text{LCM} = \frac{24 \times 44}{\text{H.C.F. of 24, 44}} = \frac{24 \times 44}{4} = 24 \times 11 = 264$

Concept Key

If there are more than 2 numbers, say 4 numbers, find the LCM of any 2 numbers and the LCM of the other 2 numbers. The LCM of their LCMs gives the LCM of all the 4 numbers.

Q. Find the LCM of 18, 28, 108, 105.

A. LCM of 18, 28 = $\dfrac{18\times 28}{\text{HCF of } 18, 28} = \dfrac{18\times 28}{2} = 18\times 14 = 252$

LCM of 108, 105 = $\dfrac{108\times 105}{\text{HCF of } 108, 105} = \dfrac{108\times 105}{3} = 108\times 35 = 3780$

Now LCM of 3780, 252 = $\dfrac{3780\times 252}{\text{HCF of } 3780, 252} = \dfrac{3780\times 252}{252} = 3780\times 1 = 3780$

So the LCM of 4 given numbers is 3780.

Shortcut method to find LCM and HCF

This method is explained using the following example.

Q. Find the LCM of 18, 108, 105

A.

2	18	108	105
2	9	54	105
3*	9	27	105
3	3	9	35
3	1	3	35
5	1	1	35
7	1	1	7
	1	1	1

Put the numbers in a row as shown above, and start dividing by the smallest prime number that divides atleast one of them.
Stop the process when all the numbers are reduced to 1.
Mark the divisors with a asterisk (*) that divides all the given numbers.
Then the LCM is the product of all the divisors.
Therefore LCM = $2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 7 = 3780$
And the HCF is given by the product of all the divisors with the asterisk (*)
Therefore HCF = 3

Concept Key

LCM and HCF of fractions :

LCM of fractions = $\dfrac{\text{LCM of numerators}}{\text{HCF of denominators}}$

HCF of fractions = $\dfrac{\text{HCF of numerators}}{\text{LCM of denominators}}$

Q. Find the LCM and HCF of $\frac{1}{2},\frac{3}{5},\frac{4}{7},\frac{5}{21}$

A. LCM of $\frac{1}{2},\frac{3}{5},\frac{4}{7},\frac{5}{21} = \dfrac{\text{LCM of 1, 3, 4, 5}}{\text{HCF of 2, 5, 7, 21}} = \dfrac{60}{1} = 60$

HCF of $\frac{1}{2},\frac{3}{5},\frac{4}{7},\frac{5}{21} = \dfrac{\text{HCF of 1, 3, 4, 5}}{\text{LCM of 2, 5, 7, 21}} = \dfrac{1}{210}$

Q. Find the greatest number that divides 204, 1190, 1445 exactly.

A. Greatest number that divides 204, 1190 and 1445 exactly is nothing but the H.C.F. of these numbers.
Factorizing the given numbers, we get
204 $= 2 \times 2 \times 3 \times 17$
1190 $= 2 \times 5 \times 7 \times 17$
1445 $= 5 \times 17 \times 17$
$\therefore$ H.C.F. = 17
Hence, the answer is 17.

Q. The H.C.F. and L.C.M. of two numbers are 38 and 98154 respectively. If one of the nos. is 1558, what is the other number?

A. Let the other number be x.
We know that, for two numbers,
Product of two numbers = L.C.M. $\times$ H.C.F. of the two numbers
$\therefore x \times 1558 = 38 \times 98154$
$\therefore x = \dfrac{38 \times 98154}{1558}$
$\therefore x = 2394$
Hence, the other number is 2394.

Relatively Prime Numbers

Two positive integers are said to be relatively prime to each other if their highest common factor is 1.
e.g. 2 and 7 are relatively prime to each other because their HCF is 1.

Concept Key

The number of ways in which a number N can be expressed as a product of two factors which are relatively prime to each other is $\boldsymbol{2^{m-1}}$ *where m is the number of different prime factors of N.*

e.g. : $N = 540 = 4 \times 27 \times 5 = 2^2 \times 3^3 \times 5^1$
$m = 3$ (i.e., 2, 3 & 5)
$\therefore$ number of ways $= 2^{3-1} = 4$
i.e., 20×27, 4×135, 108×5 and 540×1.

> **Concept Key**
>
> *If a number n is divisible by each of two relatively prime numbers then it is also divisible by the product of that two numbers.*

e.g. : If the number is 90 it is divisible by 3 and 5 both, which are relatively prime numbers.
Hence it is also divisible by $3 \times 5 = 15$.
But the same number 90 is divisible by 6 and 9 both. But it is not divisible by 54 since 6 and 9 are not relatively prime.

Indices and surds:

Indices

By a^m, we mean $a \times a \times \ldots$ m times i.e., the product of m factors each equal to a is represented by a^m. a is called the **base** and m is called the **power**. $\sqrt[n]{a}$ or $\frac{1}{a^n}$ denotes the n^{th} root of a.

> **Concept Key**
>
> ***Fundamental Laws of Indices***
>
> 1. $a^m \times a^n = a^{m+n}$
> 2. $a^m \times a^n \times a^p \times \ldots\ldots = a^{m+n+p+\ldots}$
> 3. $(a^m)^n = a^{mn}$
> 4. $a^m \div a^n = a^{m-n}$
> 5. $a^{\frac{p}{q}} = \sqrt[q]{a^p}$ *where a and p are real numbers and* $q \neq 0$.
> 6. $a^{\frac{1}{n}} = \sqrt[n]{a}$
> 7. $a^{-n} = \frac{1}{a^n}$
> 8. $a^0 = 1$
> 9. $(a \times b)^m = a^m \times b^m$ *and* $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.
> 10. *If* $a^m = a^n$ *and* $a \neq -1, 0, 1$ *then* $m = n$.

Note :

$a^{m^n} \neq \left(a^m\right)^n$.

For example, $2^{3^4} \neq \left(2^3\right)^4 \Rightarrow 2^{81} \neq 2^{3+3+3+3} \Rightarrow 2^{81} \neq 2^{12}$.

Surds

Any root of a rational number, which cannot be exactly found is called a surd (an irrational number)

e.g.: $\sqrt{2}, \sqrt[3]{4}, 2+\sqrt{2}$

For a surd, $\sqrt[n]{a}$, a is called the **radicand** and n is called the **order** of the surd. $\sqrt{\ }$ is the radical sign.

Concept Key

For $\sqrt[n]{a}$ to be surd

1. *the radicand a should be a positive rational number*
2. *n should be a natural number*
3. *$\sqrt[n]{a}$ is an irrational number.*

Concept Key

Laws of Surds

1. $\left(\sqrt[n]{a}\right)^n = a$
2. $\sqrt[n]{a} \,.\, \sqrt[n]{b} = \sqrt[n]{ab}$ *also* $\left(\sqrt[n]{a}\ \sqrt[n]{b}\right)^n = a.b$
3. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$ *also* $\left(\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}\right)^n = \dfrac{a}{b}$
4. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$
5. $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

Pure Surd

A surd which consists wholly of an irrational number is called a pure surd e.g., $\sqrt{8}, \sqrt[4]{5}$.

Mixed Surd

If a surd has one factor which is a rational number other than one and the other factor as an irrational number then the surd is called a mixed surd.

e.g., $2\sqrt{5}$, $\dfrac{5}{9}\sqrt{75}$, $-2\sqrt[3]{29}$.

Every mixed surd can be expressed as a pure surd. However, a pure surd cannot always be converted into a mixed one.

e.g., $2\sqrt[3]{2} = \sqrt[3]{2 \times 2^3} = \sqrt[3]{16}$

Note :

$\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$

Rationalization

If two surds are such that their product is rational, each of them is said to be rationalized when multiplied by the other and either of them is said to be a rationalizing factor of the other.

Q. What is the value of $(125)^{-2/3} + (1/27)^{-4/3}$

A. $$(125)^{-\frac{2}{3}} + \left(\frac{1}{27}\right)^{-\frac{4}{3}} = \left(\frac{1}{125}\right)^{2/3} + 27^{4/3}$$

$$= \left(\frac{1}{5}\right)^{3\times\frac{2}{3}} + 27^{4\times\frac{1}{3}} = \left(\frac{1}{5}\right)^{2} + (3)^{3\times4\times(1/3)}$$

$$= \left(\frac{1}{25}\right) + 3^4 = 81 + \frac{1}{25} = 81\frac{1}{25}.$$

Q. Find the value of : $\dfrac{x^{m+2n}\ x^{3m-8n}}{x^{5m-6n}}$ when x = 2, m = 2.

A. $$\frac{x^{m+2n}\ x^{3m-8n}}{x^{5m-6n}} = x^{m + 2n + 3m - 8n - (5m - 6n)}$$

$$= x^{m + 2n + 3m - 8n - 5m + 6n}$$

$$= x^{-m} = \frac{1}{x^m} = \frac{1}{2^2} = \frac{1}{4}.$$

Q. What is the value of $3^{2^3} \div (3^2)^3$?

A. $(3^2)^3 = 3^2 \times 3^2 \times 3^2 = 3^6$

and $3^{2^3} = 3^{2\times2\times2} = 3^8$

$\therefore$ the given expression $= 3^8 \div 3^6$

$= 3^{8-6} = 3^2 = 9.$

Q. Solve for n : $32^{n-2} = \dfrac{64}{8^n}$

A. $32^{n-2} = \dfrac{64}{8^n}$

$\Rightarrow 2^{5(n-2)} = 2^6 \div 2^{3n}$

$\Rightarrow 2^{5n-10} = 2^{6-3n}$

$\Rightarrow 5n - 10 = 6 - 3n$ (Comparing indices)

$\Rightarrow 8n = 16$ or $n = 2.$

Q. If $a = \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $b = \dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

Find the value of a + b

A. $a + b = \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$$= \frac{(\sqrt{5}+\sqrt{3})^2 + \left(\sqrt{5}-\sqrt{3}\right)^2}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}$$

$$= \frac{5+3+2\sqrt{15}+5+3-2\sqrt{15}}{5-3} = \frac{16}{2} = 8.$$

Squares and Square–roots:

Square of a number

If a number is multiplied by itself the product so obtained is called the square of that number. e.g., square of 7 = 7 × 7 = 49. i.e., $7^2 = 7 \times 7 = 49$.

Perfect square

The square of a natural number is called a perfect square.

Concept Key

Some properties of square numbers :

1. *A square cannot end with an odd number of zeros.*
2. *A square number cannot end with 2, 3, 7 or 8.*
3. *Every square number is a multiple of 3, or exceeds a multiple of 3 by unity.*
4. *Every square number is a multiple of 4 or exceeds a multiple of 4 by unity.*
5. *If a square number ends in 9, the preceding digit is even.*

Square of a number ending in 5 :

Multiply the number of tens by the next higher integer and annex 25 to the right of the product.
e.g., $105^2 = 11025$.

Square of a number consisting wholly of 9s :

The square is found by writing down (n – 1) nines followed by 8 and then (n – 1) zeros followed by 1 (where n is the total number of 9s in the number).
e.g. $999^2 = 998001$

Square root of a number

The square root of a given number is that whose square is equal to the given number.

Principal Square root

The equation $x^2 = 16$ is satisfied by two numbers, x = 4 and x = –4. The **positive** one, 4 is called the (principal) square root of 16 and is denoted by $\sqrt{16}$

Methods to find square root

Method 1:

To find the square root of a number by prime factorization

$\sqrt{1334025} = \sqrt{3\times3\times5\times5\times7\times7\times11\times11}$

$= 3\times 5\times 7\times 11 = 1155$

Method 2:

To find the square root of a number by division method

The given number is separated in pairs from right to left e.g., 2119936 will be grouped as 2, 11, 99, 36 and 105625 will be grouped as 10, 56, 25.

Step 1 : The integer whose square is less than or equal to the first group is written on the top and left of the number.

Step 2 : The square of the integer is subtracted from the first group.

Step 3 : The next group is brought down and written with the remainder and forms the new dividend.

Step 4 : Add the quotient to the previous quotient and put it in the divisor.

e.g. (i)

	1	4	
1	2	11	99
1			
24	1	11	
4			
28	15	99	

e.g. (ii)

	3	2	
3	10	56	25
3			
62	1	36	
2	1		
64	32	25	

Step 5 : The new divisor is obtained by writing a suitable digit say A to the right of the integer from step 4 and this number is multiplied by A.
The suitable digit is also the quotient e.g., if A =3 in above example.

(i) $23 \times 3 = 69$ which is well within 111 so try A = 4; 24 = 96 in example 2.
with A = 5; $25 \times 5 = 125$ is out of range of 111.

Step 6 :

e.g. (i)

	1456			
1	2	11	99	36
1	1			
24	111			
4	96			
285	15	99		
5	14	25		
2906	174	36		
6	174	36		
2912	0			

$\sqrt{2119936} = 1456$

e.g. (ii)

	325		
3	10	56	25
3	9		
62	1	56	
2	1	24	
645	32	25	
5	32	25	
650	0		

$\sqrt{10,56,25} = 325$

Q. Find the least number, by which 3888 is multiplied so that the resulting number is a perfect square.

A. Factorising the given number, we get

$3888 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3$

$= 2^2 \times 2^2 \times 3^2 \times 3^2 \times 3$

We are left with sigleton 3, so we must multiply by another 3 so that the resulting number will be a perfect square.

Q. Find the least number which must be subtracted from 6156 to make it a perfect square.

A. Using method 2 to find square roots, we get

	78
761	56
+7	49
1481	256
−1	184
	72

Here, we got a remainder 72. Hence 72 is the least number which must be subtracted from 6156 to make it a perfect square.

Q. What is the cost of erecting a fence round a square field of area 1000000 square meters at the rate of Rs. 10 per meter?

A. Area of the field = 1000000 sq. m.

$\therefore$ side of the field = $\sqrt{1000000} = 1000\text{m}$

$\therefore$ perimeter of the field = $4 \times 1000 = 4000$ m

Q. Find the cube root of 343.

A. $343 = 7 \times 7 \times 7$. $\therefore$ cube root of 343 = 7

Q. By what number should 21600 be multiplied to make it a perfect cube?

A. $21600 = 2^3 \times 2^2 \times 3^3 \times 5^2$

Thus 21600 should be multiplied by 2×5 to make it a perfect cube

$\therefore$ The perfect cube number = 216000

Cube root of 216000 = $2 \times 2 \times 3 \times 5 = 60$

Remainder Theorem:

Two numbers when divided by a certain divisor leave remainder r_1 and r_2 respectively. When the sum of the number is divided by the same divisor, the remainder is r_3.

and if $r_3 < r_1$ and

$r_3 < r_2$

then the theorem states that,

Divisor = $r_1 + r_2 - r_3$.

When the two given numbers are same, then

Divisor = $2r_1 - r_3$ $\because$ $r_1 = r_2$

Q. A number when divided by a certain divisor left remainder 63. When twice the given number was divided by same divisor, remainder was 55. What is the divisor?

A. Here $r_1 = r_2 = 63$, $r_3 = 55$

The required divisor is $2 \times 63 - 55$

= 126 – 55 = 71.

Fibonacci Numbers : form a sequence $\{a_n\}$ where

$a_{n+2} = a_{n+1} + a_n$ and $a_1 = 1, a_2 = 1$ e.g. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144

nth Fibonacci number Fn is given by

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

Given any Fibonacci number greater than 1, you can calculate the next Fibonacci number. Call the given number A, the next Fibonacci number is $\left[\frac{A+1+A\sqrt{5}}{2}\right]$ where bracket indicates rounding down to the nearest integer. e.g. if A = 13, next Fibonacci number is 21.54 = 21 then next number is 32.

Note:

i. In a generalized Fibonacci sequence, the sum of the first n terms is F_{n+2} minus the second term of the series.

ii. The square of any Fibonacci number differs by 1 from the product of the two Fibonacci numbers on each side.

iii. $(F_n)^2 + (F_{n+1})^2 = F_{2n+1}$

iv. For any four consecutive Fibonacci number A, B, C and D; $C^2 - B^2 = A \times D$

Short cut Methods in Multiplication and Division

1. To multiply by 9, 99, 999, 9999, etc.

9 = 10 − 1, 99 = 100 − 1, 999 = 1000 − 1, 9999 = 10000 −1

Place as many zeroes to the right of the multiplicand as there are 9's in the multiplier and from the result subtract the multiplicand itself.

e.g., (i) 26234 × 999 = 26234000 − 26234 = 26207766
(ii) 78654 × 4988

Here 4988 = 5000 − 12

78654 × 5000 = 393270000

78654 × 12 = 943848

78654 × 4968 = 393270000 − 943848 = 392326152

2. To multiply by powers of 5

To multiply by 5, place one 0 and divide by 2. e.g. $24 \times 5 = \frac{24}{2} \times 10 = 120$

To multiply by 25, place two 0's and divide by 4, e.g. $12 \times 25 = \frac{12}{4} \times 100 = 300$

To multiply by 125, place three 0's and divide by 8.

e.g. $24 \times 125 = \frac{24}{8} \times 1000 = 3000$

To multiply by 625, place four 0's and divide by 16.

e.g. $32 \times 625 = \frac{32}{16} \times 10000 = 20000$

3. To divide by powers of 5

Division by 5, 25, 125, 625 respectively, multiply by 2, 4, 8, 16 and cut off from the right 1, 2, 3, 4 digits respectively to get the quotient. To get the remainder, divide the figure cut off by 2, 4, 8, 16 respectively.

e.g. $35736 \div 25 \Rightarrow 35736 \times 4 = 142944 \Rightarrow 1429 : 44$

Quotient = 1429; Remainder $= \frac{44}{4} = 11$

4. Finding product of 2 two–digit numbers

To multiply 26 by 37, first multiply the last digits and write the units digit of this product as units digit of the final result, i.e. 7 × 6 = 42. Hence, 2 is units digit and 4 is the carry, then cross multiply, i.e. multiply 2 by 7 and 3 by 6 and add the results and also add the carry, i.e. 14 + 18 + 4 = 36. Write units digit of the sum in tens place, i.e. 6 in ten's place 3 is carry. Now multiply the first digits of the tens numbers, add the carry and write the result as first digit/s of final product, i.e. 2 × 3 = 6 and add i.e. 6 + 3 = 9. $\therefore 26 \times 37 = 962$

5. Method of finding remainder when a given number is divided by 9

Add up the digits rejecting every 9 and digits whose sum makes up 9. Also, whenever the sum is more than 9, subtract 9, and add the remainder to the next digit. The last remainder, which is left, is the '9 remainder' or '9 over'.

e.g. (i) 34578

Beginning from the units digit 7 + 8 = 15; 1 + 5 = 6
Adding 6 to the next numbers; 5 + 6 = 11; 1 + 1 = 2
Adding to the next number 4; 4 + 2 = 6
Adding to the next number 6 + 3 = 9; 9 − 9 = 0
i.e. 34578: 15, 6, 11, 2, 6, 9, 0
$\therefore$ The remainder is 0.

(ii) 98375479

Beginning from the units digits 7 + 6 = 13; 1 + 3 = 4
Adding 4 to the next digits 4; 4 + 4 = 8
Adding 8 to the next digit 5; 8 + 5 = 13; 1 + 3 = 4
Adding 4 to the next digit 7; 4 + 7 = 11; 1 + 1 = 2
Adding 2 to the next digit 3; 2 + 3 = 5
Adding 5 to the next digit 8; 5 + 8 = 13; 1 + 3 = 4
The next digit 9 is rejected
i.e. 13, 4, 8, 13, 4, 11, 2, 5, 13, 4,
$\therefore$ The remainder is 4.

The remainder that is obtained when a number is divided by 9 is also called the digital root of that number.

Concept Key

OBSERVE

Cast out 8 from nine digits in order and multiply the remaining number by multiples of 9. Observe the result.

12345679 × 9 = 111111111
12345679 × 18 = 222222222
12345679 × 27 = 333333333
12345679 × 36 = 444444444
12345679 × 45 = 555555555
12345679 × 54 = 666666666
12345679 × 63 = 777777777
12345679 × 72 = 888888888
12345679 × 81 = 999999999

Q. Arrange the fractions $\frac{2}{15}, \frac{3}{10}, \frac{5}{21}$ in the ascending order of their magnitude.

A.

5	15,	10,	21
3	3,	2,	21
3	1,	2,	7

L.C.M. of 15, 10, 21 = 5 × 3 × 2 × 7 = 210

$\frac{2}{15} = \frac{2 \times 14}{15 \times 14} = \frac{28}{210}$, ...(1)

$\frac{3}{10} = \frac{3 \times 21}{10 \times 21} = \frac{63}{210}$, ...(3)

$\frac{5}{21} = \frac{5 \times 10}{21 \times 10} = \frac{50}{210}$, ...(2)

The factors 14, 21, and 10 are obtained by dividing 210 by the denominators of the given fractions in turn. By comparing the numerators we see that $\frac{28}{210}$ is the least and $\frac{63}{210}$ the greatest of the–given fractions.

Thus $\frac{2}{15}, \frac{5}{21}, \frac{3}{10}$ are the original fractions when arranged in ascending order.

(i) To find the square root of a positive decimal fraction:

If the given number is in the form of a decimal fraction, the digits to the left of the decimal point and those to the right of the decimal point are grouped independently. In the integral part, groups of two digits each are formed, starting from the decimal point and moving to the left.

In the fractional part, starting from the decimal point the groups are formed towards the right.

e.g.,(i)

	2 3.2 5
2	540.56 25
2	4
43	140
3	129
462	1156
2	924
4645	23225
5	23225
4650	0

e.g.,(ii)

	107.5	
1	11556.25	
	1	
	20	15
	0	0
	207	1556
	7	1449
	2145	10725
	5	10725
	2150	0

1 = 1 and 9^2 = 81. So the square root of a one digit or a 2 digit number is always a one digit number 10^2 = 100 and 99^2 = 9801. So the square root of a three digit number or a four digit number is always a two digit number.

Thus, the number of a digit in the square root of a ‘n’ digit number is

(i) $\frac{n+1}{2}$, where n is odd.

(ii) $\frac{n}{2}$, where n is even.

(ii) Square root of fraction:

The square root of a fraction, if the denominator is a perfect square, is found by taking the square root of the numerator and denominator separately. e.g. $\sqrt{\frac{2}{49}} = \frac{1}{2}\sqrt{2}$

In case of a mixed fraction, it must first be expressed as an improper fraction.

In the case of a fraction whose denominator is not a perfect square. We may either convert the fraction into a decimal as a first step, or multiply the numerator and denominator by a number that will make the denominator a perfect square.

Concept Key

SQUARE PYRAMIDS

1 × 1	=	1
11 × 11	=	121
111 × 111	=	12321
1111 × 1111	=	1234321
11111 × 11111	=	123454321
111111 × 111111	=	12345654321
1111111 × 1111111	=	1234567654321
11111111 × 11111111	=	123456787654321
111111111 × 111111111	=	12345678987654321

LOGARITHMS

If a is a positive real number, other than 1 and $a^m = x$, then m is called the logarithm of x to the base a, written as $\log a^x$.

Thus, $a^n = x \Rightarrow \log_a x = n$

$10^4 = 10000 \Rightarrow \log_{10} 10000 = 4$

i) $a \log_a x = x$, $\log_a 1 = 0$
ii) If $0<a<1$ and $0<x<1$ then $\log_a x > 0$
iii) If $a > 1$ and $0<x<1$ then $\log_a x < 0$

Laws of logarithms

1) $\log_a(xy) = \log_a x + \log_a y$
2) $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
3) $\log_a(x^k) = k \log_a x$, $\log_a(x^{1/k}) = \frac{1}{k} \log_a x$
4) $\log_a a = 1$

5) $\log_a N = \dfrac{1}{\log_N a}$

6) $\log_a N = \dfrac{\log_b N}{\log_b a}$

Given an equation $\log_a M = \log_b N$

i) If M = N, then a will be equal to b

ii) If a = b, then M will be equal to N.

Solved Examples :

Q.1 Simplify $\dfrac{3\frac{1}{3}\times 2\frac{1}{4}}{3\frac{1}{3}-2\frac{1}{4}} \div \dfrac{5\frac{1}{7}}{2\frac{3}{5}}$

A. $\dfrac{3\frac{1}{3}\times 2\frac{1}{4}}{3\frac{1}{3}-2\frac{1}{4}} \div \dfrac{5\frac{1}{7}}{2\frac{3}{5}}$

$= \dfrac{\frac{10}{3}\times\frac{9}{4}}{\frac{10}{3}-\frac{9}{4}} \div \dfrac{\frac{36}{7}}{\frac{13}{5}} = \dfrac{\frac{10\times 9}{3\times 4}}{\frac{40-27}{3\times 4}} \div \left(\dfrac{36}{7}\div\dfrac{13}{5}\right)$

$= \dfrac{10\times 9}{13} \div \dfrac{36\times 5}{7\times 13} = \dfrac{10\times 9}{13}\times\dfrac{7\times 13}{36\times 5} = \dfrac{2\times 7}{4} = \dfrac{1}{2}\times 7$

$= \dfrac{7}{2}$

$= 3\dfrac{1}{2}$

Q.2 Simplify : 4.7 × 13.23 + 4.7 × 9.43 + 4.7 × 77.34

[**Note :** x × a + x × b + x × c = x × (a + b + c)]

A. 4.7 × (13.23 + 9.43 + 77.34) = 4.7 × 100 = 470.

Q.3 Simplify : $\dfrac{1}{3\frac{1}{5}} - \dfrac{2\frac{1}{4}}{9} + \dfrac{3\frac{5}{8}}{2} + \dfrac{\frac{4}{7}}{4\frac{4}{7}}$

A. $\dfrac{1}{\frac{16}{5}} - \dfrac{\frac{9}{4}}{9} + \dfrac{\frac{29}{8}}{2} + \dfrac{\frac{4}{7}}{\frac{32}{7}}$

$= \dfrac{5}{16} - \left(\dfrac{9}{4}\times\dfrac{1}{9}\right) + \left(\dfrac{29}{8}\times\dfrac{1}{2}\right) + \left(\dfrac{4}{7}\times\dfrac{7}{32}\right)$

$= \dfrac{5}{16} - \dfrac{1}{4} + \dfrac{29}{16} + \dfrac{1}{8} = \dfrac{5-4+29+2}{16} = \dfrac{32}{16} = 2.$

Q.4 If 1 < x < 5 and 3 < y < 8, what is the largest integer value of x + y ?

A. If x and y are integers, the largest value would be 4 + 7 = 11. But, if it is not mentioned, the numbers are always real, Though here we want the largest integer value of x + y, x & y can be real numbers.
If x = 4.7 and y = 7.3, we get
x + y = 4.7 + 7.3 = 12 which is the largest possible integer value of x + y.
Note :
Also (x = 4.5, y = 7.5), (x = 4.4, y = 7.6) …. will get the same answer.

Q.5 A number is "fantastic" if it is a multiple of 2, 3 and 4. How many fantastic numbers are there between –100 and 100.

A. A number is fantastic if it is a multiple of 2, 3 and 4 i.e. it should be the multiple of L.C.M. of 2, 3 and 4.
L.C.M. of (2, 3, 4) = 12
∴ The required nos. are 12, 24, 36, 48, 60, 72, 74, 96 and their opposites and 0.
∴ Total fantastic nos. = 8 + 8 + 1 = 17

Q.6 If 0 < x < 1, which of the following lists the numbers in increasing order ?
i) $2\sqrt{x}, 2x, 2x^2$ ii) $3x, 3x^2, 3\sqrt{x}$ iii) $4x^2, 4x, 4\sqrt{x}$
iv) $5x, 5\sqrt{x}, 5x^2$ v) $x^2, \sqrt{x}, x$

A. For any number x, between 0 and 1
$x^2 < x$ and $x < \sqrt{x}$
$\therefore x^2 < x < \sqrt{x}$
The multipliers in the option doesn't make any difference to the order.
If $x^2 < x < \sqrt{x}$, then
$4x^2 < 4x < 4\sqrt{x}$ (multiplying all terms by a +ve number)
∴Ans is option (iii)

Q.7 Find the total number of factors of 76230

A. Factorising 76230, we get
$76230 = 2 \times 3 \times 3 \times 5 \times 7 \times 11 \times 11$
$= 2^1 \times 3^2 \times 5^1 \times 7^1 \times 11^2$
∴Total number of factors = $(1+1)(2+1)(1+1)(1+1)(2+1) = 2 \cdot 3 \cdot 2 \cdot 2 \cdot 3 = 72$

Q.8 Three students A, B and C buy a set of books. A pays 4/15^th^ of the cost, B pays 3/5^th^ of the cost and C pays the rest which is equal to 100 Rs. What is the cost of the set ?

A. Let 'x' Rs. be the cost of the set.
∴C must pay, $x - \frac{4x}{15} - \frac{3x}{5} = \frac{15x}{15} - \frac{4x}{15} - \frac{9x}{15} = \frac{2x}{15}$ Rs.
But C paid Rs. 100
$\therefore \frac{2x}{15} = 100$
$\therefore x = \frac{100}{2} \times 15 = 50 \times 15 = 750$ Rs.
∴Cost of the set is 750 Rs.

Q.9 A number when divided by 145 leaves the remainder 58. what is the remainder if it is divided by 29.

A. Let the number be x

Then x ÷ 145 leaves the remainder 58

$\therefore x = 145y + 58$

$= 29\ (5y) + 29\ (2)$

$x = 29\ (5y + 2)$

Hence, x is totally divisible by 29.

$\therefore$ Remainder = 0

Q.10 For any positive integer n, τ (n) represents the number of positive divisors of n. What is the value of $\tau(\tau(\tau(\tau(24))))$?

A. $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3^1$

$\therefore$ Positive divisor of 6 = $(3 + 1)\ (1 + 1) = 4 \times 2 = 8$

$\therefore \tau(24) = 8$

$\therefore \tau(\tau(\tau(\tau(24))))$

$= \tau(\tau(\tau(8)))$

$= \tau(\tau(4))$

$= \tau(3) = 2.$

Q.11 What fractional part of the week is 96 hours?

A. In a week, there are 7 days, and in a day there are 24 hours.

$\therefore$ In a week, there are 7×24 hours

$\therefore$ Fractional part of the week $= \dfrac{96}{\text{Total hours in a week}} = \dfrac{96}{7 \times 24} = \dfrac{24 \times 4}{7 \times 24} = \dfrac{4}{7}$

Q.12 $\dfrac{4}{17}$ of 24 is equal to $\dfrac{8}{51}$ of what number.

A. Let the number be x.

$\therefore \dfrac{4}{17} \times 24 = \dfrac{8}{51} x$

$\therefore x = \dfrac{51}{8} \times \dfrac{4}{17} \times 24 = 36$

Q.13 A wealthy businessman gave away 1/2 of his wealth to the first son; 1/3 of the remaining to the second son; 1/2 of the remaining to his third son; and the rest to his youngest son. If the youngest son got Rs.6 lakhs. What is the total wealth of the business ?

A. By given the youngest son gets

$\left(\dfrac{1}{2}\right)\left(1 - \dfrac{1}{3}\right)\left(\dfrac{1}{2}\right)$

i.e. $\dfrac{1}{2} \times \dfrac{2}{3} \times \dfrac{1}{2} = \dfrac{1}{6}$ of the total wealth

$\therefore$ But youngest son gets 6 lakhs.

i.e. $\dfrac{1}{6}$th of the total wealth is 6 lakhs.

$\therefore$ Total wealth = 6×6 = 36 lakhs.

Q.14 Find the greatest number that will divide 119, 201 and 239 leaving remainders 2, 3 and 5 respectively.

A. Required number is the H.C.F. of (119 − 2), (201 − 3) and (239 − 5)
i.e. H.C.F. of 117, 198, 234
117 = ③ × ③ × 13
198 = 2 × ③ × ③ × 11
234 = 2 × ③ × ③ × 13
∴ H.C.F. = 3 × 3 = 9
∴ The required number is 9.

Q.15 Find the least number which when divided by 12, 15, 20 and 35 leaves the remainder 6 in each case.

A. Required number = (L.C.M. of 12, 15, 20 and 35) + 6 = 420 + 6 = 426

Q.16 Find the smallest number which when increased by 5 is divisible by each of 24, 32, 36 and 54.

A. Required number = (L.C.M. of 24, 32, 36 and 54) − 5
Finding L.C.M. of 24, 32, 36 and 54 :

2	24	32	36	54
2	12	16	18	27
2	6	8	9	27
2	3	4	9	27
2	3	2	9	27
3	3	1	9	27
3	1	1	3	9
3	1	1	1	3
	1	1	1	1

∴ L.C.M. = 2 × 2 × 2 × 2 × 2 × 3 × 3 × 3 = 864
∴ Required number = 864 − 5 = 859

Q.17 Find the length of the largest possible square slab which can be used in paving the floor 5 m 44 cm long and 3m 74 cm broad? Also find the number of slabs required.

A. Length and Breadth in centimeters are
544 cm and 374 cm
∵ The slabs are to be square in shape, so its sides must be equal and as we need the largest square slab, its side must be the highest that fits exactly along length and breadth.
∴ Length of the side of this square
= H.C.F. of 544 cm and 374 cm = 34 cm
Now we want to find the number of such slabs in paving the floor;
Number of slabs = area of the floor ÷ area of one slab.

$$= \frac{544 \times 374}{34 \times 34} = 16 \times 11 = 176.$$

Q.18 Sum of two numbers is 670 and their G.C.M., is 67. How many such pairs of numbers are possible?

A. Since the numbers are multiples of their G.C.M. so let the two numbers be 67x and 67y where x and y have no common factor i.e. they are relatively prime.

Now $67x + 67y = 670$ or $x + y = 10$

Possible values of x and y so that $x + y = 10$ are

(1, 9), (2, 8), (3, 7), (4, 6), (5, 5)

As (2, 8), (4, 6), and (5, 5) are not relatively prime to each other.

$\therefore$ Only two numbers are possible namely (1, 9) and (3, 7) where

$x = 1$ and $y = 9$; $x = 3$ and $y = 7$.

Hence the numbers are (i) 67×1 and 67×9 i.e., 67 and 603

(ii) 67×3 and 67×7 i.e., 201 and 469

Q.19 In a school 437 boys and 342 girls have been divided into the largest possible separate classes, so that in each class number of boys is same as number of girls. What is the number of classes ?

A. Here, we should have the maximum number of students in a class. Also, number of boys in a class is equal to number of girls in a class.

$\therefore$ Number of boys or Number of girls in each class is

H.C.F. (437, 342) = 19

$\therefore$ Number of classes = $\frac{437}{19} + \frac{342}{19} = 23 + 18 = 41$classes

i.e. 23 classes for boys and

18 classes for girls.

Q.20 A gardner planted saplings in such a way that evey row had as many samplings as every column. If in all there are 841 saplings, how many samplings were there in each row ?

A. Let, the number of samplings in each row and column = x

$\therefore$ Total number of samplings = $x^2 = 841$

$\therefore x = \sqrt{841} = 29$

Q.21 Find the number whose square is equal to the difference between the squares of 6467 and 4683.

A. Let x be the required number, so that x^2 is equal to

$(6467)^2 - (4683)^2 = (6467 + 4683)(6467 - 4683)$

$x^2 = 11150 \times 1784$

$= 2 \times 5 \times 5 \times 223 \times 2 \times 2 \times 2 \times 223$

$= 2^2 \times 2^2 \times 5^2 \times 223^2$

$\therefore$ square root of $x^2 = 2 \times 2 \times 5 \times 223 = 4460$.

Q.22 Find the square root of $\frac{0.081}{0.0064} \times \frac{0.484}{6.25}$

A. Making equal decimal places in the numerator and denominator.

Q.23 Simplify: $\cfrac{1}{2-\cfrac{2}{2+\cfrac{4}{2-\cfrac{5}{2+\cfrac{6}{7}}}}}$

A. $\cfrac{1}{2-\cfrac{2}{2+\cfrac{4}{2-\cfrac{5}{2+\cfrac{6}{7}}}}}$

$= \cfrac{1}{2-\cfrac{2}{2+\cfrac{4}{2-\cfrac{35}{20}}}}$ $= \cfrac{1}{2-\cfrac{2}{2+\cfrac{80}{5}}}$

$= \cfrac{1}{2-\cfrac{2}{2+\cfrac{}{16}}}$ $= \cfrac{1}{2-\cfrac{2}{18}}$ $= \cfrac{1}{2-\cfrac{1}{9}}$ $= \cfrac{1}{\cfrac{17}{9}} = \frac{9}{17}$

Q.24 Simplify: $\dfrac{.45\times[.73-\{(.35\,\text{of}\,.9\div.15\,\text{of}\,4.2-.05)-.4\}]}{1.53\times.35}\div\dfrac{0.25}{0.315}$

A. $\dfrac{0.45\times[0.73-\{(0.35\text{ of }09\div0.15\text{ of }4.2-0.05)-0.4\}]}{1.53\times0.35}\div\dfrac{0.25}{0.315}$

$= \dfrac{0.45\times\left[0.73-\left\{\left(\dfrac{0.35\times.9}{0.15\times4.2}-0.05\right)-0.4\right\}\right]\times\dfrac{0.315}{0.25}}{1.53\times0.35} = \dfrac{0.45\times[0.73-\{0.05\}]\times0.315}{1.53\times0.35\times0.25}$

$= 0.72$

Q.25 (i) if X381 is divisible by 11, find the value of smallest natural number X ?
(ii) if 381Y is divisible by 9, find the value of smallest natural number Y?

A. (i) X 381 is the number given to us. If this number is divisible by 11 then the divisibility rule for 11 must be satisfied i.e. (X + 8) – (3 + 1) = 0 or a multiple of 11 i.e. $x + 4 = 11 \Rightarrow x = 7$

(ii) 381Y is the number given to us. If this number is divisible by 9 then the divisibility rule for 9 must be satisfied i.e. (3 + 8 + 1 + Y) must be divisible by 9 i.e. $(12 + Y) = 18 \Rightarrow Y = 6$

Q.26 What will be the remainder obtained when $(1234567890123456789)^{24}$ is divided by 6561?

A. (1234567890123456789) is the number given to us. Sum of all the digits = 2(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) = 2 (45) = 90 i.e. this number is divisible by 9. Thus we can write

$(1234567890123456789)^{24}$ as $(9K)^{24}$. Similarly 6561 can be written as 9^4. This means that the given number will be completely divisible by 6561 and remainder will be 0.

Q.27 Find the smallest natural number 'n' such that n! is divisible by 990?

A. Since $990 = 2 \times 3^2 \times 5 \times 11$, n! must contain a factor of 11. Since 11 is a prime number, it must itself be contained in the product, so n = 11 is the smallest possible value.

Q.28 What is the remainder if $1237 \times 1239 \times 1241 \times 1243$ is divided by 30?

A. r_1, r_2, r_3, r_4 be the reminders when 1237, 1239, 1241, 1243 divided by 30 respectively. The reminder will be the reminder which we will have by dividing the product of reminders with the number given, 7, 9, 11, 13 are the reminders and when $7 \times 9 \times 11 \times 13$ divided by 30 will have 9 as reminder.

$$\sqrt{\frac{0.0810}{0.0064} \times \frac{0.484}{6.250}} = \sqrt{\frac{810}{64} \times \frac{484}{6250}} = \sqrt{\frac{81}{64} \times \frac{484}{625}} = \frac{9}{8} \times \frac{22}{25} = \frac{99}{100} = .99.$$

Q.29 If L.C.M. of two natural numbers is 36 and their sum is 30. Find the two numbers.

A. The two natural numbers could be any two of the factors of 36 viz.
1,2,3,4,6,9,12,18.
But only 12 + 18 = 30. Hence the required numbers are 12 and 18.

Q.30 If $(\log_3 x)(\log_x 2x)(\log_{2x} y) = \log_x x^2$, then y equals

A. $$\frac{\log x}{\log 3} \times \frac{\log 2x}{\log x} \times \frac{\log y}{\log 2x} = \frac{2 \log x}{\log x}$$

$$\frac{\log y}{\log 3} = 2$$

$y = 9$

Q.31 If $\dfrac{\log_2\left(9 - 2^x\right)}{3 - x} = 1$, then the value of x is

A. $\log_2(9 - 2^x) = 3 - x,\ x \neq 3$

$\Rightarrow 9 - 2^x = 2^{3-x} = 2^3 \times 2^{-x}$

$\Rightarrow 9 - 2^x - 2^{2x} = 2^3$

$\Rightarrow 2^{2x} - 9.2^x + 8 = 0$

$\Rightarrow (2^x - 8)(2^x - 1) = 0$

$\Rightarrow 2^x = 8 = 2^3$

$\Rightarrow x = 3$

$\Rightarrow 2^x = 1 = 2^0$

$\Rightarrow x = 0$

Since, $x \neq 3$, therefore x = 0.

Q.32 There are 408 boys and 312 girls in a school which are to be divided into equal sections of either boys or girls alone. Find the maximum number of boys or girls that can be placed in a section. Also find the total number of sections thus formed.

A. HCF of 408 & 312 =24
maximum no. of students in a group = 24
Total number of sections = (408 + 312)/24 = 30

Q.33 Solve for x : log 2x – log 6 = log 12

A. log2x = log 6 + log 12
log 2x = log 72
2x = 72
x = 36

Q.34 A wine seller has three different types of wine, 403 gallon of 1st type, 434 gallon of 2nd type, 465 gallon of 3rd type. Find the least possible number of casks of equal size in which different types of wines can be filled without mixing.

A. HCF of 403, 434, 465 is 31
wine 1 can be stored in 13 casks of 31 gallon size
wine 2 can be stored in 14 casks of 31 gallon size
wine 3 can be stored in 15 casks of 31 gallon size
Total no. of casks = 13 + 14 + 15 = 42

Q.35 A, B, and C start at the same time from the same place in the same direction to walk round a circular course 12 miles long. A, B and C walk respectively at the rate 3, 7 and 13 miles per hour. In what time will they come together again at starting point?

A. Time for one revolution .

Time required for $A = \frac{12}{3} = 4$ hour,

Time required for $B = \frac{12}{7}$ hours,

Time required for $C = \frac{12}{13}$,

Required time = L.C.M. $(4, \frac{12}{7}, \frac{12}{13})$ = 12 hrs.

Q.36 The L.C.M. of two numbers is 28 times of their H.C.F. The sum of their L.C.M. and H.C.F. is 1740. If one of the numbers is 240, find the other number.

A. L + H = 1740. L = 28H.
H = 60 and L = 1680
$1680 = 60(f1 \times f_2)$
$240 = 60 \times f_1$. Therefore, $f_1 = 4$ and $f_2 = 7$. Hence, the other number is 60×7 = 420.

Q.37 Find x if $[2^{x-1} \times 4^{2x+1}] / [8^{x-1}] = 64$

A. $[2^{x-1} . 4^{2x+1}] / 8^{x-1} = 64$
L.H.S. – $[2^{x-1}.(2^2)^{2x+1}] / (2^3)^{x-1} = [2^{x-1}.2^{4x+2}] / 2^{3x-3} = 2^{5x+1} / 2^{3x-3} = 2^{2x+4}$
R.H.S. $64 = 2^6$
$\therefore\ 2^{2x+4} = 2^6$
$\therefore\ 2x + 4 = 6$
$\therefore\ x = 1.$

Q.38 Find the least number, which must be added to 36520 to make it exactly divisible by 187.

A. 1 and 8 are the first two digits of the divisor and 3 and 6, those of the dividend.
Hence we try $200 \times 187 = 37400$.
Subtracting 374 twice from this, we get 36652.
We confirm that (36652–36520).
Also, 36652–36520 =132.<187.
Hence the least number that should be added is 132.

Q.39 If $\frac{x^2}{yz}+\frac{y^2}{xz}+\frac{z^2}{xy}=3$, find $x+y+z$.

A. Multiplying both sides by xyz, we get,
$x^3+y^3+z^3=3xyz. \Rightarrow (x^3+y^3+z^3-3xyz)=0$
$\therefore (x+y+z)(x^2+y^2+z^2-xy-yz-zx)=0$ Hence answer cannot be determined.
The result is True for all numbers where x = y = z.

Q.40 Solve $\log\frac{5}{4}+\log 14-\log\frac{7x}{3}=-1$

A. $\log\left(\frac{5}{4}\times 14\times\frac{3}{7x}\right)=-1$

$\log\frac{15}{2x}=-1$ (Here base is 10)

$\frac{15}{2x}=10^{-1}=\frac{1}{10}$

$x=\frac{15\times 10}{2}=75$

Q.41 If $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$, and a + b + c = 5, find the value of $a^3+b^3+c^3-3abc$.

A. Multiplying both sides of the first equation by abc, we get,
bc + ac + ab = 0 …..(1)
Also, $(a + b + c)^2 = 5^2$
$a^2+b^2+c^2+2(ab+bc+ac)=25$ …..(2)
Substituting (1) in (2).
$a^2+b^2+c^2=25$
We have, $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-ac-bc)$
L.H.S. = (5) (25–0)
$a^3+b^3+c^3-3abc=125.$

Q.42 What will be the remainder obtained when $(9^6 + 1)$ is divided by 8?

A. (x^n-y^n) is always divisible by (x–y). We can write (9^6+1)as$(9^6-1+1+1)$ ie.
$[(9^6-1^6)+2]$.
Now (9^6-1^6) is completely divisible by 8. So when $[(9^6-1^6)+2]$ is divided by 8 remainder will be 2 only.

Q.43 Find the last digit of the number $(7)^{71}$?

A. This question can be easily solved using the concept of power cycle.
$7^1 = 7$, digit at unit's place is 7.
$7^2 = 49$, digit at unit's place is 9.
$7^3 = 343$, digit at unit's place 3.
$7^4 = 2401$, digit at unit's place is 1.
Now after this depending on the power of 7 the digits at unit's place will be repeated in a cycle i.e. for 7^5 digit at unit's place will be again 7, for 7^6 digit at unit's place will be 9 and so on.
In order get 7^{71} one has take 17 complete cycles (68/4,as 68 is the highest multiple of 4 less than 71) and then 3 more places i.e. unit's digit will be same as 7^3. Hence the answer is 3.

Q.44 Which of the following is greater 2^{300}, 3^{200}, or 5^{150}?

A. $2^{300} = (2^3)^{100} = 8^{100}$

$3^{200} = (3^2)^{100} = 9^{100}$

$5^{150} = (5\sqrt{5})^{100}$

Now in all the 3 cases the powers are same so the number with highest base will be the greatest, hence the answer is $(5\sqrt{5})^{100}$ as $\sqrt{5} > 2$ so $5\sqrt{5} > 9$.

Q.45 Evaluate $\log_{10} 200 + \log_{10} 40 + 3\log_{10} 25$

A. $\log_{10}200 + \log_{10}40 + 3\log_{10}25$
$= \log_{10}(2 \times 10^2) + \log_{10}(2^2 \times 10) + 3\log_{10}5^2$
$= \log_{10}2 + 2\log_{10}10 + 2\log_{10}2 + \log_{10}10 + 6\log_{10}5$
$= 3 + 3\log_{10}2 + 6 - 6\log_{10}2$
$= 9 - 3\log_{10}2$

Directions for (Qs.46 to 48) : Priyanka was studying at 11m, Ahmedabad. Her father was interested in puzzles. So she sent a note to him asking for money the note said.

$$\begin{array}{r} \text{N E E D} \\ + \text{ M O R E} \\ \hline \text{M O N E Y} \\ \hline \end{array}$$

Where each alphabet represents single – digit numbers from 0 to 9 and MONEY is a five digit number.

Q.46 How much 'Money' did her father send to her ?
(A) 10000 (B) 10999
(C) 11100 (D) 19999

Q.47 What is the value of NEED ?
(A) 9999 (B) 9900
(C) 9990 (D) 9000

Q.48 What is the value of MORE ?
(A) 1000 (B) 1900
(C) 1990 (D) 1009

Solutions for (Qs. 46 to 48) :

```
   N E E D
 + M O R E
 ---------
 M O N E Y
 ---------
```

MONEY being a five–digit number which is sum of two four digit numbers, M must be 1.
So, N must be 9 and O must be zero.
Going along E = N = 9 and R must be 0.
Also, E = Y = 9 and D = 0.
So

```
  9990
 +1009
 ------
 10999
 ------
```

Q.49 $\sqrt{2}, \sqrt[3]{4}$ and $\sqrt[4]{6}$ in ascending order are :

A. Given surds are of order 2, 3, 4 whose 1.c.m is 12.
Changing each one of given surds to that of order 12, we get :

$$\sqrt{2} = 2^{\frac{1}{2}} = 2^{\left(\frac{1}{2}\times\frac{6}{6}\right)} = (2^6)^{1/12} = (64)^{1/12}$$

$$\sqrt[3]{4} = 4^{\frac{1}{3}} = 4^{\left(\frac{1}{3}\times\frac{4}{4}\right)} = 4^{\frac{4}{12}} = \left(4^4\right)^{\frac{1}{12}} = (256)^{\frac{1}{12}}$$

$$\sqrt[4]{6} = 6^{\frac{1}{4}} = 6^{\left(\frac{1}{4}\times\frac{3}{3}\right)} = 6^{\frac{3}{12}} = \left(6^3\right)^{\frac{1}{12}} = (216)^{\frac{1}{12}}$$

Now, $(64)^{\frac{1}{12}} < (216)^{\frac{1}{12}} < (256)^{\frac{1}{12}}$

$\therefore \sqrt{2} < \sqrt[4]{6} < \sqrt[3]{4}$.

SERIES :

Turn odd man out : As the phrase speaks itself, in this type of problems, a set of numbers is given in such a way that each one, except one satisfies a particular definite property. The one which does not satisfy that characteristic is to be taken out.

Insert the missing number : In this type of problems, a set of numbers is given such that one of the no. is missing. You have to identify the missing number.

Turn odd man out.

Q.1 56, 72, 90, 110, 132, 150

(A) 72 (B) 110
(C) 132 (D) 150

A. The nos. are 7×8, 8×9, 9×10, 10×11, 11×12
12×13. So, 150 is wrong.

Q.2 3, 5, 7, 12, 17, 19

(A) 19 (B) 17

(C) 13 (D) 12

A. Each of the nos except 12 is a prime number.

Q.3 19, 28, 39, 52, 67, 84, 102

(A) 52 (B) 102

(C) 84 (D) 67

A. The pattern is $x^2 + 3$ where x = 4, 5, 6, 7, 8, 9 etc. But 102 is out of pattern.

Q.4 1, 2, 6, 15, 31, 56, 91

(A) 31 (B) 91

(C) 56 (D) 15

A. Add $1^2, 2^2, 3^2, 4^2, 5^2, 6^2$

So, 91 is wrong.

Q.5 10, 14, 28, 32, 64, 68, 132

(A) 32 (B) 68

(C) 132 (D) 28

A. Alternatively we add 4 and double the next.

So 132 is wrong.

Q.6 2, 9, 28, 65, 125, 216, 344

(A) 2 (B) 28

(C) 65 (D) 216

A. $2 = (1^3 + 1); 9 = (2^3 + 1); 28 = (3^3 + 1); 65 = (4^3 + 1);$

$125 = (5^3 + 1); 216 \neq (6^3 + 1)$ & $344 = (7^3 + 1)$.

$\therefore$ 216 is a wrong number.

Q.7 7, 8, 18, 57, 228, 1165, 6996

(A) 8 (B) 18

(C) 57 (D) 228

A. Let the given numbers be A, B, C, D, E, F, G. Then,

A, A × 1, B × 2 + 2, C × 3 + 3, D × 4 + 4, E × 5 + 5, F × 6 + 6 are the required numbers.

Clearly, 228 is wrong.

Insert the missing number

Q.8 16, 33, 65, 131, 261, (––––)

(A) 523 (B) 521

(C) 613 (D) 721

A. Each no. is twice the preceding one with 1 added or substracted alternatively. So the next no. is (2 × 261 + 1) = 523.

Q.9 10, 5, 13, 10, 16, 20, 19, (–––––)

(A) 22 (B) 40

(C) 38 (D) 23

A. There are two series (10, 13, 16, 19) and (5, 10, 20, 40) one increasing by 3 and other multiplied by 2.

Q.10 445, 221, 109, 53, 25, (––––)

(A) 11 (B) 15
(C) 10 (D) 20

A. Go on subtracting 3 and divding the result by 2 to obtain the next number. So the number is 11.

Q.11 40960, 10240, 2560, 640, (––––)

(A) 150 (B) 200
(C) 160 (D) 225

A. Go on dividing by 4 to get next number.

Q.12 1, 8, 27, 64, 125, 216, (––––)

(A) 354 (B) 343
(C) 392 (D) 245

A. Numbers are $1^3, 2^3, 3^3, 4^3, 5^3, 6^3$
So the missing number is $7^3 = 343$.

Q.13 3, 7, 6, 5, 9, 3, 12, 1, 15, (––––)

(A) 18 (B) 13
(C) –1 (D) 3

A. There are two series, beginning respectively with 3 and 7. In one 3 is added and in another 2 is subtracted. The next number is $1 - 2 = -1$.

Q.14 2, 5, 9, 19, 37, …….

(A) 76 (B) 74
(C) 75 (D) None of these

A. Second number is one more than twice the first; third number is one less than twice the second; fourth number is one more than twice the third; fifth number is one less than the fourth.
So, the missing number is 75.

Assignment :

Q.1 When 24563 is divided by a certain divisor, quotient is 79 and remainder is 73. What is the divisor ?

(A) 315 (B) 310
(C) 325 (D) 320

Q.2 Solve : $\dfrac{6}{1+\dfrac{6}{1+\dfrac{6}{1+\ldots\infty}}}$.

(A) 2 (B) 6
(C) 3 (D) 4

Q.3 $\dfrac{8.14\times 8.14-6.13\times 6.13}{8.14-6.13} = ?$

(A) 14.27 (B) 16.27
(C) 15.72 (D) 14.72

Q.4 Simplify : $\dfrac{2.5\times2.5\times2.5-1.5\times1.5\times1.5}{2.5\times2.5+2.5\times1.5+1.5\times1.5}$

(A) 7 (B) 3
(C) 1 (D) 2

Q.5 Simplify : $\dfrac{1.2\times1.2\times1.2+2.3\times2.3\times2.3}{1.2\times1.2-1.2\times2.3+2.3\times2.3}$

(A) 4.5 (B) 3.5
(C) 5.5 (D) 6.5

Q.6 There are three gears making 60, 36 and 24 rotations per minute. There is a red spot on the circumference of each of the gears. In the beginning, all the three red spots are together. After how much time will all the spots come together again ?

(A) 6s (B) 10s
(C) 5s (D) 7s

Q.7 Suppose we have two iron rods of lengths 30 cm and 18 cm. What is the length of the largest rod by which we can measure each of these rods an exact number of times ?

(A) 6 cm (B) 8 cm
(C) 11 cm (D) 5 cm

Q.8 The HCF of two numbers is 13 and their LCM is 819. If one number is 91, find the other number.

(A) 119 (B) 117
(C) 120 (D) 115

Q.9 The difference between a two-digit number and the number obtained by interchanging the digits is 72. What is the difference between the digits of the number?

(A) 7 (B) 4
(C) 8 (D) 9

Q.10 A number, when divided by 552 gives a remainder 53. When it is divided by 23, the remainder will be

(A) 0 (B) 6
(C) 2 (D) 8

Q.11 The LCM of 25, 1.5, 1.75 and 0.015 is

(A) 1500 (B) 5.25
(C) 52.5 (D) 525

Q.12 The LCM of two numbers is 12000 and their GCM is 480. If one of the numbers is 1440, then the other number is

(A) 400 (B) 160
(C) 4000 (D) 40000

Q.13 If x + 3y = 5 and $\dfrac{x}{y}$ = 2, then y is equal to

(A) 1 (B) 0.5
(C) 2 (D) 8

Q.14 Simplify $\sqrt{14+6\sqrt{5}}$

(A) $2+2\sqrt{5}$ (B) $2+\sqrt{5}$

(C) $3+\sqrt{5}$ (D) $3+2\sqrt{5}$

Q.15 Which of the following options is smallest ?

(A) 2^{400} (B) 3^{300}

(C) 4^{180} (D) 8^{150}

Q.16 The value of $\dfrac{(0.96)^3-(0.1)^3}{(0.96)^2+0.096+(0.1)^2}$ is:

(A) 0.86 (B) 0.95

(C) 0.97 (D) 1.06

Q.17 The value of $\left(\dfrac{0.051\times 0.051\times 0.051+0.041\times 0.041\times 0.041}{0.051\times 0.051-0.051\times 0.041+0.041\times 0.041}\right)$ is:

(A) 0.00092 (B) 0.0092

(C) 0.092 (D) 0.92

Q.18 $\dfrac{(0.6)^4-(0.5)^4}{(0.6)^2+(0.5)^2}$ is equal to:

(A) 0.1 (B) 0.11

(C) 1.1 (D) 11

Q.19 8.7-[7.6-{6.5-(5.4-4.3-2)}] is simplified to:

(A) 2.5 (B) 3.5

(C) 4.5 (D) 5.5

Q.20 The value of $\left[35.7-\left(3+\dfrac{1}{3+\dfrac{1}{3}}\right)-\left(2+\dfrac{1}{2+\dfrac{1}{2}}\right)\right]$ is:

(A) 30 (B) 34.8

(C) 36.6 (D) 41.4

Q.21 The value of $\left(0.\overline{2}+0.\overline{3}+0.\overline{4}+0.\overline{9}+0.\overline{39}\right)$ is :

(A) $0.\overline{57}$ (B) $1\dfrac{20}{33}$

(C) $2\dfrac{1}{3}$ (D) $2\dfrac{13}{33}$

Q.22 The value of $2.1\overline{36}$ is :

(A) $\dfrac{47}{220}$ (B) $\dfrac{68}{495}$

(C) $2\dfrac{3}{22}$ (D) None of these

Q.23 What is the unit digit in $\left(7^{95} - 3^{58}\right)$?

(A) 0 (B) 4
(C) 6 (D) 7

Q.24 What is the unit digit in $(4137)^{754}$?

(A) 1 (B) 3
(C) 7 (D) 9

Q.25 If the number 481*673 is completely divisible by 9, then the smallest whole number in place of * will be:

(A) 2 (B) 5
(C) 6 (D) 7

Q.26 If the number 97215*6 is completely divisible by 11, then the smallest whole number in place of * will be:

(A) 3 (B) 2
(C) 1 (D) 5

Q.27 If 60% of $\frac{3}{5}$ of a number is 36, then the number is :

(A) 80 (B) 100
(C) 75 (D) 90

Q.28 The difference between a positive proper fraction and its reciprocal is $\frac{9}{20}$. The fraction is:

(A) $\frac{3}{5}$ (B) $\frac{3}{10}$
(C) $\frac{4}{5}$ (D) $\frac{5}{4}$

Q.29 The sum of the two numbers is 12 and their product is 35. What is the sum of the reciprocals of these numbers?

(A) $\frac{12}{35}$ (B) $\frac{1}{35}$
(C) $\frac{35}{8}$ (D) $\frac{7}{32}$

Q.30 If a and b are odd numbers, then which of the following is even?

(A) a + b (B) a + b + 1
(C) ab (D) ab + 2

Q.31 (51 + 52 + 53 + … + 100) = ?

(A) 2525 (B) 2975
(C) 3225 (D) 3775

Q.32 The L.C.M. of $2^3 \times 3^2 \times 5 \times 11$, $2^4 \times 3^4 \times 5^2 \times 7$ and $2^5 \times 3^3 \times 5^3 \times 7^2 \times 11$ is:

(A) $2^3 \times 3^2 \times 5$ (B) $2^5 \times 3^4 \times 5^3$
(C) $2^3 \times 3^2 \times 5 \times 7 \times 11$ (D) $2^5 \times 3^4 \times 5^3 \times 7^2 \times 11$

Q.33 The H.C.F. of $\frac{2}{3}, \frac{8}{9}, \frac{64}{81}$ and $\frac{10}{27}$ is:

(A) $\frac{2}{3}$ (B) $\frac{2}{81}$
(C) $\frac{160}{3}$ (D) $\frac{160}{81}$

Q.34 The L.C.M. of $\frac{1}{3}, \frac{5}{6}, \frac{2}{9}, \frac{4}{27}$ is:

(A) $\frac{1}{54}$ (B) $\frac{10}{27}$
(C) $\frac{20}{3}$ (D) None of these

Q.35 Three different containers contain 496 litres, 403 litres and 713 litres of mixtures of milk and water respectively. What biggest measure can measure all the different quantities exactly?

(A) 1 litre (B) 7 litres
(C) 31 litres (D) 41 litres

Q.36 The least number, which when divided by 12, 15, 20 and 54 leaves in each case a remainder of 8, is:

(A) 504 (B) 536
(C) 544 (D) 548

Q.37 The least number, which when divided by 48, 60 72, 108 and 140 leaves 38, 50, 62, 98 and 130 as remainders respectively, is:

(A) 11115 (B) 15110
(C) 15120 (D) 15210

Q.38 Six bells commence tolling together and toll at intervals of 2, 4, 6, 8, 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together?

(A) 4 (B) 10
(C) 15 (D) 16

Q.39 Four different electronic devices make a beep after every 30 minutes, 1 hour, $1\frac{1}{2}$ hour and 1 hour 45 minutes respectively. All the devices beeped together at 12 noon. They will again beep together at:

(A) 12 midnight (B) 3 a.m.
(C) 6 a.m. (D) 9 a.m.

Q.40 A, B, and C start at the same time in the same direction to run around a circular stadium. A completes a round in 252 seconds, B in 308 seconds and C in 198 seconds, all starting at the same point. After what time will they meet again at the starting point?

(A) 26 minutes 18 seconds (B) 42 minutes 36 seconds
(C) 45 minutes (D) 46 minutes 12 seconds

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