Strength of Materials

INTRODUCTION

Strength of materials is the scientific area of applied mechanics for the study of the strength of engineering materials and their mechanical behaviour in general (such as stress, deformation, strain and stress-strain relations). Strength is considered in terms of **compressive strength**, **tensile strength**, and **shear strength**, namely the limit states of compressive stress, tensile stress and shear stress respectively. Certain mechanical properties are needed to understand the basics.

MECHANICAL PROPERTIES OF MATERIALS

Strength

Strength is the property that enables a metal to resist deformation under load. The ultimate strength is the maximum strain a material can withstand. Tensile strength is a measurement of the resistance to being pulled apart when placed in a tension load.

Elasticity

Elasticity is the property of an object or material which causes it to be restored to its original shape after distortion. It is said to be more elastic if it restores itself more precisely to its original configuration. A perfectly elastic material is one which returns to its original shape and size after removal of an applied stress. An inelastic material becomes permanently deformed following application of a stress.

Isotropic Material

In this type of material, the elastic constants are the same in all directions so if a specimen is cut from a bulk material, the direction in which it is cut has no affect on the values. This applies to most metals with no pronounced grain structure.

Orthotropic Material

In this type of material, the elastic constants have different values in the x, y and z directions so the results obtained in a test depend upon the direction in which the specimen was cut from the bulk material. This applies to materials with grain structures such as wood or rolled metals.

Non-Isotropic Material

In this type of material, the elastic constants are unpredictable and the results from any two tests are never the same. This applies to materials such as glass and other ceramics.

Hardness

Hardness is the property of a material that enables it to resist plastic deformation, usually by penetration.

Malleability

Malleability is the property of being physically malleable; the property of something that can be worked or hammered or shaped under pressure without breaking.

Plasticity

Plasticity is the property of a material, such as an adhesive, which permits continuous and permanent deformation without failure upon the application of a force which exceeds the yield value of the material.

Toughness

Toughness is the property that enables a material to withstand shock and to be deformed without rupturing.

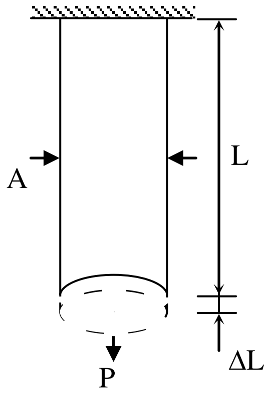
Fatigue

Fatigue strength is the ability of material to resist various kinds of rapidly changing stresses and is expressed by the magnitude of alternating stress for a specified number of cycles.

STRESS AND STRAIN

Stress

It is the internal distribution of forces within a body that balance and react to the loads applied on it.



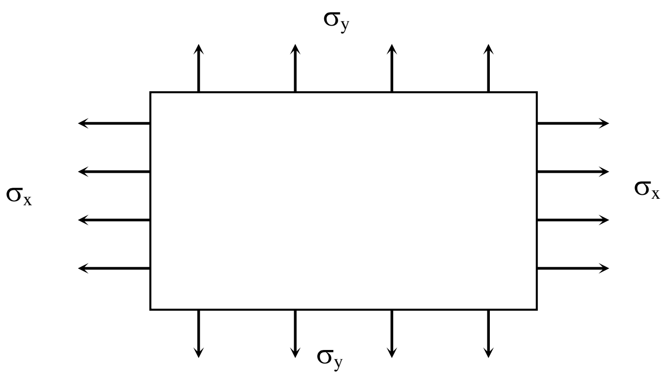
The elongation of a bar is proportional to the tensile force applied on it. This force creates internal forces in the bar to maintain equilibrium. In discussing the magnitude of internal forces let us imagine the bar cut into two parts by a cross-section and let us consider the equilibrium of the lower portion of the bar. At the lower end of this portion the tensile force P is applied.

On the upper end the forces represent the action of the particles of the upper portion of the strained bar on the particles of the lower portion. These forces are continuously distributed over the cross-section. In handling such continuously distributed forces, the intensity of force, i.e. the force per unit area is very important. The resultant of these forces will pass through the centroid of the cross-section and will act along the axis of the bar. Taking into account that the sum of the forces, from the condition of equilibrium, must be equal to P and denoting the force per unit of cross-sectional area by σ, we obtain,

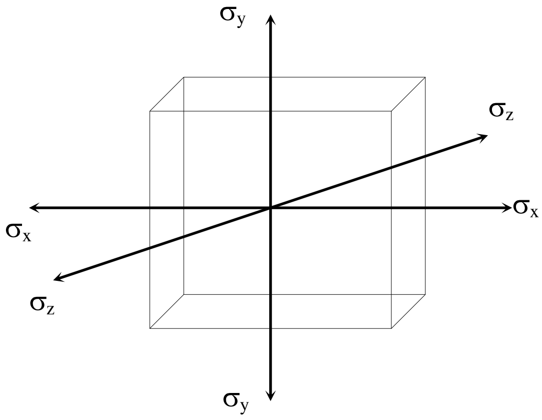
$$\sigma = \frac{P}{A}$$

This force per unit area is called unit tensile stress or simply stress.
Stress would either be one – dimensional, two dimensional or three dimensional.

1D Stress



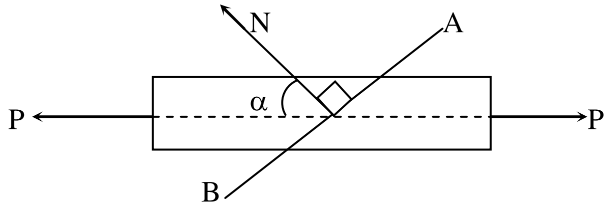
2D Stress



3D Stress

Direct and Shear Stress

Let us take a prismatic bar acted upon by an axial tension P.



Let us take a section AB inclined to the plane of the axis. The internal stresses at the section balance the external force P applied at the left end. The resultant of the forces distributed over the cross-section.

AB is equal to P. Taking the area of cross-section of the bar normal to the axis as S and α be the angle between the x – axis and plane AB, then the stress over the cross-section σ is,

$$\sigma = \frac{P}{S}\cos\alpha = \sigma_x \cos\alpha$$

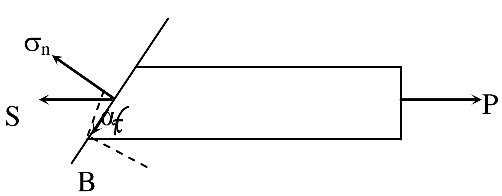
where $\sigma_x = \frac{P}{S}$ is the stress on the cross-section normal to the axis of the bar. We can see that the stress σ over any inclined cross-section of the bar is smaller than the stress σ_x over the cross-section normal to the axis of the bar. For $\alpha = \pi/2$, the stress becomes zero.

This stress component σ_x, is parallel to the axis of the bar and is resolved into two components.
The stress component σ_n perpendicular to the cross-section is called the normal or direct stress.

$$\sigma_n = \sigma\cos\alpha = \sigma_x \cos^2\alpha$$

The tangential component τ is called shearing stress and has the value,

$$\tau = \sigma\sin\alpha = \sigma_x \cos\alpha \sin\alpha = \frac{\sigma_x}{2}\sin 2\alpha$$



It is seen that the normal stresses σ_n produce extension of the element in the direction of the normal to the cross-section AB and the shearing stresses produce sliding of section AB.

From the above equations we see that the maximum normal stress acts on cross-sections normal to the axis of the bar and we get,

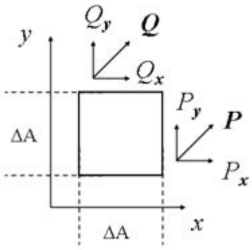
$$(\sigma_n)_{max} = \sigma_x$$

The maximum shearing stress, acts on cross-sections inclined at 45° to the axis of the bar, where $\sin 2\alpha = 1$, and has the magnitude

$$(\tau)_{max} = \frac{1}{2}\sigma_x$$

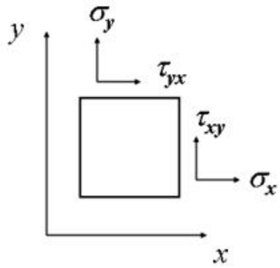
Although the maximum shearing stress is one half the maximum normal stress, this stress is sometimes the controlling factor when considering the ultimate strength of materials which are much weaker in shear than in torsion.

Plane stress is a two-dimensional state of stress in a body. This is a good model when a flat thin body is loaded in the plane of the body. A small volume element in equilibrium experiences forces in balance. By specifying some co-ordinates, the forces ***P*** and ***Q*** can be resolved normal and perpendicular to the faces of the volume element.



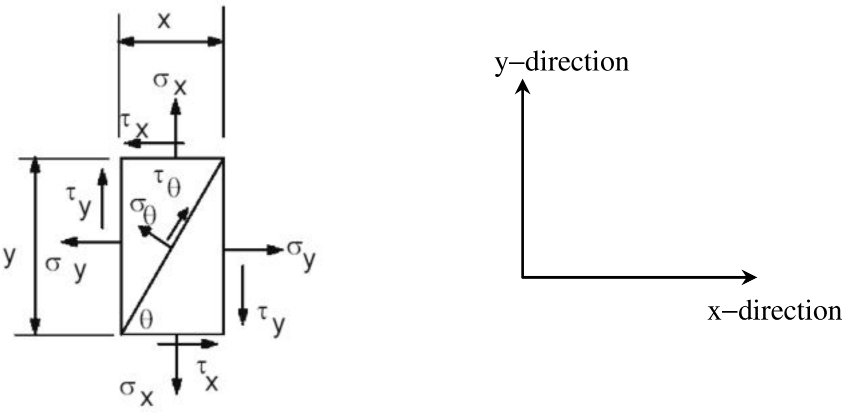
The stresses on the element are:

- Normal stresses:
  → $\sigma_x = P_x/\Delta A$
  → $\sigma_y = Q_y/\Delta A$
- Shear stresses:
  → $\tau_{xy} = P_y/\Delta A$
  → $\tau_{yx} = Q_x/\Delta A$



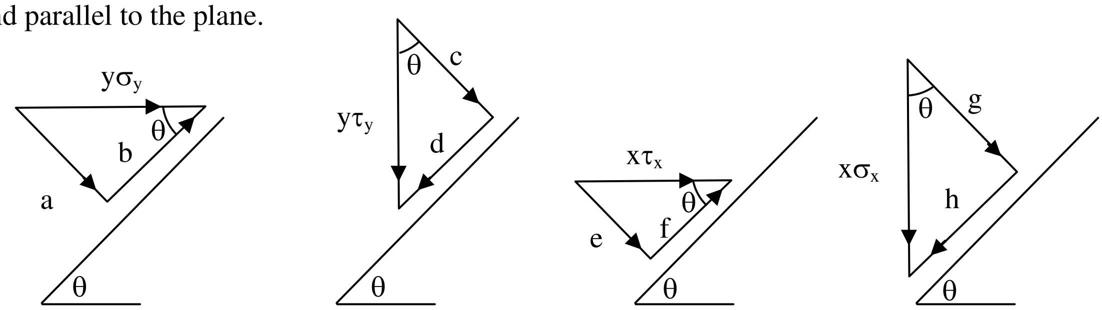
Materials in a stressed component often have direct and shear stresses acting in two or more directions at the same time. This is a complex stress situation.

Consider a rectangular part of the component material as shown in the figure below. Stress σ_x acts on the x plane and σ_y acts on the y plane. The shear stress acting on the plane on which σ_x acts is τ_x and τ_y acts on the plane on which σ_y acts. The shear stresses are complementary and so must have opposite rotation. We will take clockwise shear to be positive and anti-clockwise as negative.



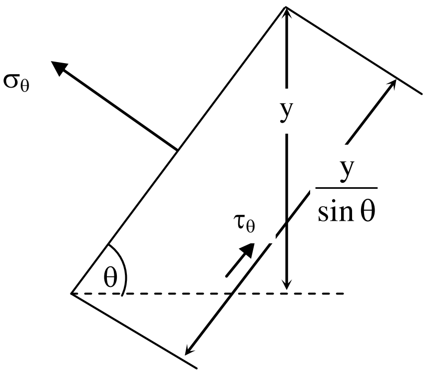
Now consider a plane at angle θ to the x plane. The plane is in equilibrium so all the forces and moments on the plane must add up to zero. The stresses must first be turned into forces. If the material is 1 m thick normal to the paper then the areas are x and y on the sides and y/sinθ or x /cosθ on the sloping plane.

Consider the balance of forces on the right lower corner. First the forces are resolved perpendicular and parallel to the plane.



$$a = y\sigma_y \sin\theta \qquad b = y\sigma_y \cos\theta$$
$$c = y\tau_y \cos\theta \qquad d = y\tau_y \sin\theta$$
$$e = x\tau_x \sin\theta \qquad f = x\tau_x \cos\theta$$
$$g = x\sigma_x \cos\theta \qquad h = x\sigma_x \sin\theta$$

All the forces normal to the plane must add up to (y/sinθ) σ_θ.



Balancing normal forces we have a + c + e + g = (y/sinθ) σ_θ, and
All the forces parallel to the plane must add up to (y/sinθ) τ_θ
- f + h + b - d = (y/sinθ) τ_θ

Making the substitutions and conducting algebraic process will yield the following results.

$$\frac{y}{\sin\theta}\sigma_\theta = y\sigma_y \sin\theta + y\tau_y \cos\theta + x\tau_x \sin\theta + x\sigma_x \cos\theta$$

$$\sigma_\theta = \sigma_y \sin^2\theta + \tau_y \sin\theta\cos\theta + \frac{x}{y}\tau_x \sin^2\theta + \frac{x}{y}\sin\theta\cos\theta$$

$$\sigma_\theta = \sigma_y \frac{1-\cos 2\theta}{2} + \tau_y \frac{\sin 2\theta}{2} + \tau_x \frac{\sin^2\theta}{\tan\theta} + \sigma_x \frac{\sin\theta\cos\theta}{\tan\theta}$$

$$\sigma_\theta = \sigma_y \frac{1-\cos 2\theta}{2} + \tau_y \frac{\sin 2\theta}{2} + \tau_x \frac{\sin 2\theta}{2} + \sigma_x \frac{1+\cos 2\theta}{2}$$

$$\sigma_\theta = \frac{\sigma_y}{2} - \sigma_y \frac{\cos 2\theta}{2} + \tau_y \frac{\sin 2\theta}{2} + \tau_x \frac{\sin 2\theta}{2} + \frac{\sigma_x}{2} + \sigma_x \frac{\cos 2\theta}{2}$$

The shear stress on both planes are equal so denote them both by $\tau_{xy} = \tau_x = \tau_y$. Hence,

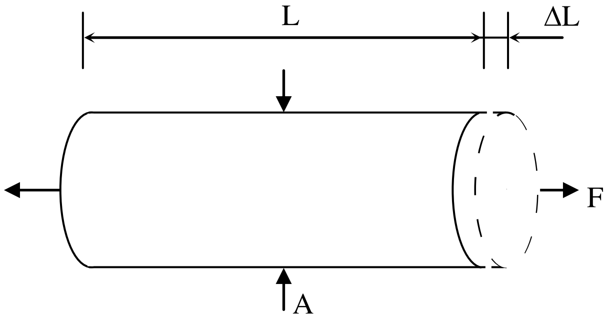
$$\sigma_\theta = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_x - \sigma_y)\cos 2\theta}{2} + \tau_{xy}\sin 2\theta \qquad \ldots(1.1)$$

Repeating the process for the shear stress we get

$$\tau_\theta = \frac{(\sigma_x - \sigma_y)\sin 2\theta}{2} - \tau_{xy}\cos 2\theta \qquad \ldots(1.2)$$

Strain

It is the geometrical expression of deformation caused by the action of stress on a physical body. Strain therefore expresses itself as a change in size and/or shape. If strain is equal over all parts of the body, it is referred to as *homogeneous* strain; otherwise, it is *inhomogeneous* strain.

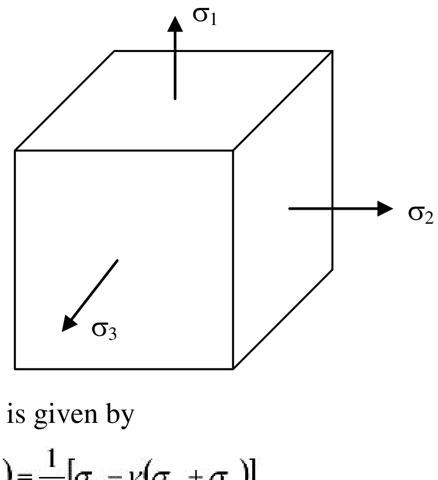


$$\text{strain} = \frac{\text{extension}}{\text{length}} = \frac{\Delta L}{L}$$

Strain has no units because it is the ratio of two lengths.

Relationship between Principal Stress & Strain

In any stress system, there are 3 mutually perpendicular planes on which only direct stress acts and there is no shear stress. These are the principal planes and the stresses are the principal stresses. These are designates σ1, σ2 and σ3. The corresponding strains are the principal strains ε1, ε2 and ε3.



The strain in each direction is given by

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2 - \nu\sigma_3) = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)]$$

$$\varepsilon_2 = \frac{1}{E}(\sigma_2 - \nu\sigma_1 - \nu\sigma_3) = \frac{1}{E}[\sigma_2 - \nu(\sigma_1 + \sigma_3)]$$

$$\varepsilon_3 = \frac{1}{E}(\sigma_3 - \nu\sigma_1 - \nu\sigma_3) = \frac{1}{E}[\sigma_3 - \nu(\sigma_1 + \sigma_2)]$$

where ν = Poisson's ratio.

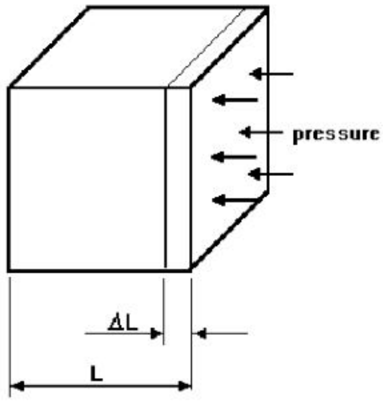
$\therefore$ Volumetric strain = $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = (\sigma_1 + \sigma_2 + \sigma_3)\left[\frac{1-2\nu}{E}\right]$

Volumetric Strain

Consider a cube of a certain material which is stressed in the x-direction by a compressive pressure as shown in the figure alongside.

The change in volume is $L^2\,\Delta L$.

Now consider that the cube is strained by an equal amount in the y and z-directions also. With very little error the total change in volume is $3L^2\Delta L$. The original volume is L^3.



When a solid object is subjected to a pressure p such that the volume is reduced, the volumetric strain is ε_v = change in volume/original volume.

In the case of a cube this becomes

$$\varepsilon_v = 3L^2\frac{\Delta L}{L^3} = \frac{3\Delta L}{L}$$

$$\varepsilon_v = 3\,\varepsilon$$

ε is the equal strain in all three directions.

In case of a beam, volumetric strain is given by

$$\varepsilon_{vol} = \frac{P}{E}\left(\frac{1-2}{1/\nu}\right)$$

where P = load

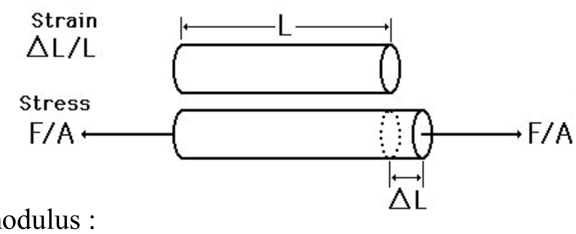
ν = Poisson's ratio

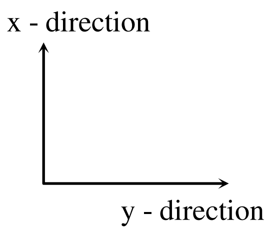
STRESS-STRAIN RELATIONSHIPS AND ITS CONSTANTS

Young's Modulus

For the description of the elastic properties of linear objects like wires, rods, columns which are either stretched or compressed, a convenient parameter is the ratio of the stress to the strain, a parameter called the Young's modulus of the material.

Young's modulus can be used to predict the elongation or compression of an object as long as the stress is less than the yield strength of the material.





Young's modulus :

$$E = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta L/L}$$

Hooke's Law

Hooke's law simply states that the extension of a spring (or any other stretchable object) is directly proportional to the force acting on it.

Stress $\propto$ Strain

This law is only true if the elastic limit of the object has not been reached.

If the elastic limit has been reached the object will not return to its original shape and may eventually break.

Bulk Modulus of Elasticity

The *Bulk Modulus of Elasticity* is the elastic response to hydrostatic pressure and equilateral tension or the volumetric response to hydrostatic pressure and equilateral tension. It is also the property of a material that determines the elastic response to the application of stress.

Poisson's Ratio

Poisson's Ratio is the ratio of transverse strain to corresponding axial strain on a material stressed along one axis.

Consider a piece of material in 2-dimensions. The stress in the y direction is σ_y and there is no stress in the x–direction. When it is stretched in the y–direction, it causes the material to get thinner in all the other directions at right angles to it. This means that a negative strain is produced in the x–direction. For elastic materials, it is found that the applied strain (ε_y) is always directly proportional to the induced strain (ε_x) and its ratio is called Poisson's Ratio.

$$\frac{\varepsilon_x}{\varepsilon_y} = -\nu$$

The strain produced in the x–direction is $\varepsilon_x = -\nu\varepsilon_y$

If stress is applied in x–direction then the resulting strain in the y–direction would similarly be $\varepsilon_y = -\nu\varepsilon_x$

The resulting strain in any one direction is the sum of the strains due to the direct force and the induced strain from the other direct force.

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\sigma_y = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right) \qquad \text{...(1A)}$$

Similarly $$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\sigma_x\right) \qquad \text{...(1B)}$$

Converting Strain into Stress

We have already derived

$$\varepsilon_1 = \frac{\sigma_1 - \nu\sigma_2}{E}$$

$$\varepsilon_2 = \frac{\sigma_2 - \nu\sigma_1}{E}$$

Rearrange to make σ_2 the subject.

$$\sigma_2 = \varepsilon_2 E + \nu\sigma_1$$

Substitute this into the first formula

$$\varepsilon_2 = \frac{\left(\varepsilon_2 E + \nu\sigma_1\right) - \nu\sigma_1}{E}$$

Rearrange

$$\sigma_1 = \left(\frac{E}{1-\nu^2}\right)\left(\varepsilon_1 + \nu\varepsilon_2\right)$$

If we do the same but make σ_2 the subject of the formula we get

$$\sigma_2 = \left(\frac{E}{1-\nu^2}\right)\left(\varepsilon_2 + \nu\varepsilon_1\right)$$

Relationship between the elastic constants

When the material is compressed by a pressure p the stress is equal to -p because it is compressive. The bulk modulus is then

$$K = \frac{-p}{\varepsilon_v}$$

$$K = \frac{E}{3(1-2\nu)}$$

This shows the relation between E, K and ν.
The relation between E, G and ν is given by

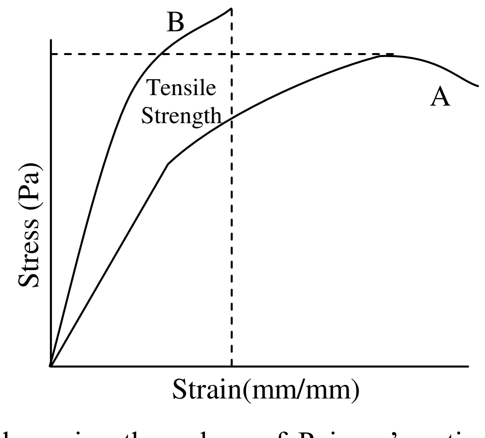
$$G = \frac{E}{2(1+\nu)}$$

where E = Young's modulus
G = Bulk modulus
ν = Poisson's ratio

TENSILE TESTING

In the tensile test, a piece of ductile material is subjected to gradually increasing pull in a tension test machine.

The figure below represents a stress strain diagram in which the stress is plotted against strain.
The straight line in the beginning represents that part of the test where the extension remains proportional to the load and is termed as *line of proportionality.* Hooke's law is valid for this range. The stress at the point where this line ends is called the *proportional limit.* If the load is increased beyond this, elongation increases more rapidly and the elongation becomes permanent.



A = Ductile material
B = Brittle material, ceramics & glass
Tensile strength – the maximum stress applied to the specimen.

The tables below give the values of Poisson's ratio and Modulus of Elasticity of some common materials.

Material	**Poisson's ratio, ν**
Al & its alloys	0.33
Brass	0.33
Bronze	0.33
Bronze porous	0.22
Copper	0.33
Iron, cast	0.26
Iron, wrought	0.30
Magnesium alloys	0.33
Steel	0.30
Zinc alloys	0.27

Polymer :

Nylons	0.40
Polyethylene	0.35
Rubber	0.50

Ceramics :

Alumina	0.28
Cemented carbide	0.19
Silicon carbide	0.19
Silicon nitride	0.26

Material	**Modulus of Elasticity, E (MPa)**
Al and Al–alloys	62 & 70
Babbitt lead based and tin based	29 & 52
Brasses	100
Bronzes	100 to 117
Bronze porous	60
Copper	124
Iron, grey cast	109
Iron, malleable cast	170
Iron, wrought	170
Steel, low alloys	196
Steel, medium high alloys	200
Steel stainless	193
Steel high speed	212
Zinc alloy	50

Polymer :

Acetal	2.7
Nylon	1.9
Polyethylene	0.9
Natural rubber	0.004

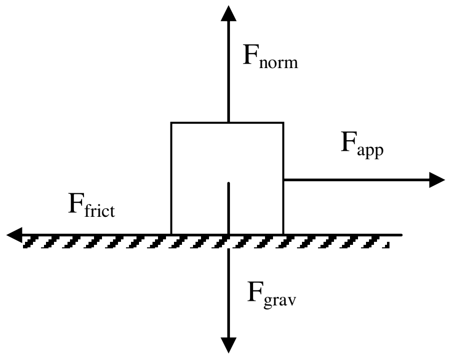
Ceramic :

Alumina	390
Graphite	27
Cemented carbide	450
Silicon carbide	450
Silicon nitride	314

The highest Poisson’s ratio is 0.5 for rubber and the lowest is 0.19 for silicon carbide. Poisson’s ratio cannot be zero.

FREE BODY DIAGRAMS

Free-body diagrams are diagrams used to show the relative magnitude and direction of all forces acting upon an object in a given situation.

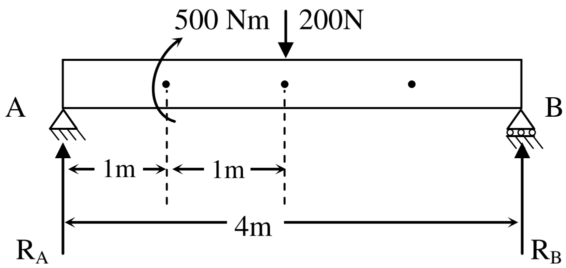


where,

F_{grav} : Gravitational force.
F_{norm} : Normal force acting due to reaction from ground.
F_{app} : Applied Force
F_{frict} : Frictional Force between object and ground.

Example :

For a loading of a beam as given below, the reaction at support A is



Solution :

R_A, R_B are the reactions.

Now doing vertical force balance,

$$R_A + R_B = 200 \text{ N}$$

Doing moment balance about B,

$$R_A \times 4 + 500 = 200 \times 2$$

$$R_A = -25 \text{ N} \quad \text{acting downwards.}$$

Example :

In the above problem, the reaction at support B is ?

Solution

As calculated before,

$$R_A = -25 \text{ N}$$

$$\therefore \quad R_B = (200+25)\text{N} = 225\text{N}\uparrow$$

MOHR'S CIRCLE FOR PRINCIPAL STRAIN AND STRESS

Determining principal stresses and planes

The stresses are a maximum or minimum on the principal planes so using the maximum or minimum theory we have

$$\frac{d\sigma_\theta}{d\theta} = -(\sigma_x - \sigma_y)\sin 2\theta + 2\tau\cos 2\theta = 0$$

$$2\tau\cos 2\theta = (\sigma_x - \sigma_y)\sin 2\theta$$

$$\tan 2\theta = \frac{2\tau}{\sigma_x - \sigma_y}$$

There are two solutions to this equation giving answers less than 360 degrees and they differ by 90 degrees. From this, the angle of principal plane may be found. From this the principal stresses are calculated and are found to be

$$\sigma_{max} = \sigma_1 = \frac{(\sigma_x + \sigma_y)}{2} + \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

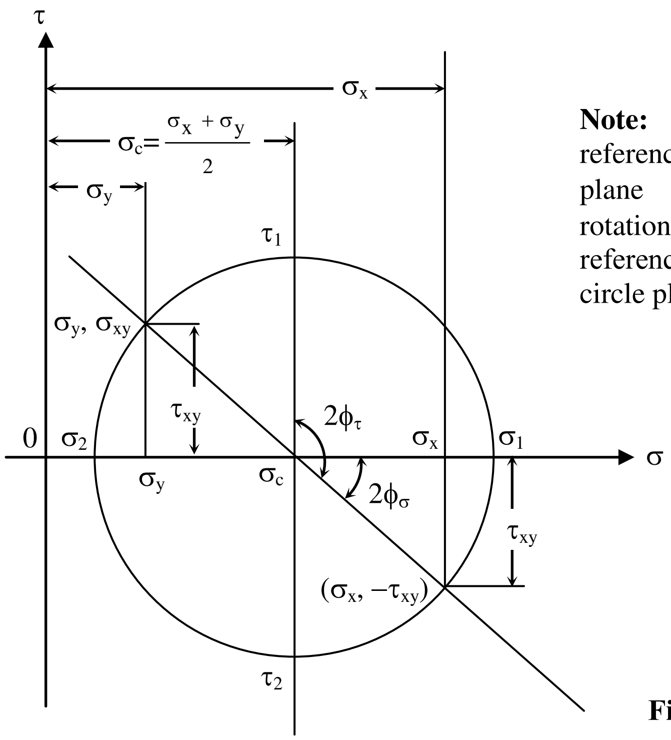
$$\sigma_{min} = \sigma_2 = \frac{(\sigma_x + \sigma_y)}{2} - \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

Similarly the shear stress is found to be

$$\tau_{max} = \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{min} = \frac{-\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2} = -\left[\frac{\sigma_1 - \sigma_2}{2}\right]$$

Mohr's Circle illustrates principal strain and strain transformations via a graphical format,



Note: A rotation, from a reference stress state in the real plane ϕ corresponds to a rotation of 2ϕ from the reference points in the Mohr's circle plane.

Fig. Mohr's Circle

Centre of Mohr's circle is placed at $\left(\dfrac{\sigma_x + \sigma_y}{2}, 0\right)$

The radius of circle = r = $\sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$

Principal stresses have values $\sigma_{1,2}$ = centre ± radius
The maximum shear stress = radius

The normal strains are equal to the principal strains when the element is aligned with the principal directions, and the shear strain is equal to the maximum shear strain when the element is rotated 45° away from the principal directions.

As the element is rotated away from the principal (or maximum strain) directions, the normal and shear strain components will always lie on Mohr's Circle.

To establish Mohr's Circle, we first recall the strain transformation formulas for plane strain at a given location,

$$\begin{cases} \varepsilon_{x'} - \dfrac{\varepsilon_x + \varepsilon_y}{2} = \dfrac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \varepsilon_{xy}\sin 2\theta \\ \varepsilon_{x'y'} = -\dfrac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \varepsilon_{xy}\cos 2\theta \end{cases}$$

Using basic trigonometric relations to combine the two above equations we have,

$$\left(\varepsilon_{x'} - \frac{\varepsilon_x + \varepsilon_y}{2}\right)^2 + \varepsilon_{x'y'}^{\ 2} = \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \varepsilon_{xy}^{\ 2}$$

This equation is an equation for a circle. To make this more apparent, we can rewrite it as,

$$\left(\varepsilon_{x'} - \varepsilon_{Avg}\right)^2 + \varepsilon_{x'y'}^{\ 2} = R^2$$

where,

$$\varepsilon_{Avg} = \frac{\varepsilon_x + \varepsilon_y}{2} \quad , \qquad R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \varepsilon_{xy}^{\ 2}}$$

The circle is centered at the average strain value ε_{Avg}, and has a radius *R* equal to the maximum shear strain,

The Mohr's Circle for plane stress can also be obtained from similar procedures.

SHEAR FORCE AND BENDING MOMENT DIAGRAMS

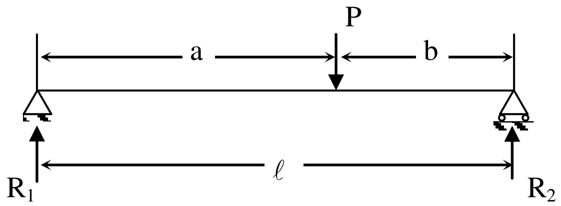
Types of Beams and Loads :

Point Loads
A point load is shown as a single arrow and acts at a point.

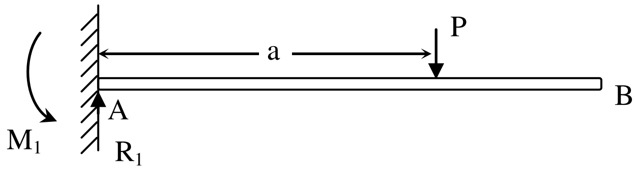
Uniform Loads
Uniform loads are shown as a series of arrows and has a value of w N/m. For any given length x meters, the total load is wx Newton and this is assumed to act at the centre of that length.

Beams

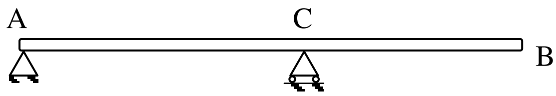
Here, we discuss the simplest types of beams having a vertical plane of symmetry through the longitudinal axis. It is assumed that all applied forces are vertical and act in the plane of symmetry so that bending occurs in the same plane.



The above figure represents a beam with simply supported ends. Points of support A and B are hinged so that the ends of the beam can rotate freely during bending.



The above figure represents a cantilever beam. The end A of this beam is built into the wall and cannot rotate during bending, while the end B is totally free.



The above figure represents a beam with an overhanging end. This beam is hinged to an immovable support at the end A and rests on a movable support at C.

All the above three cases represent statically determinate beams since the reactions at the supports produced by a given load can be determined from the equations of statics. For the simply supported beam carrying a vertical load P, we see that the reaction R_2 at the end B must be vertical, since this end is free to move horizontally. Then from the equations of statics, $\Sigma X = 0$, it follows that reaction R_1 is also vertical. The magnitudes of R_1 and R_2 are then determined from the equations of moments. Equating to zero the sum of the moments of all forces with respect to point B, we obtain

$$R_1\ell - Pb = 0$$

from which,

$$R_1 = \frac{Pb}{\ell}$$

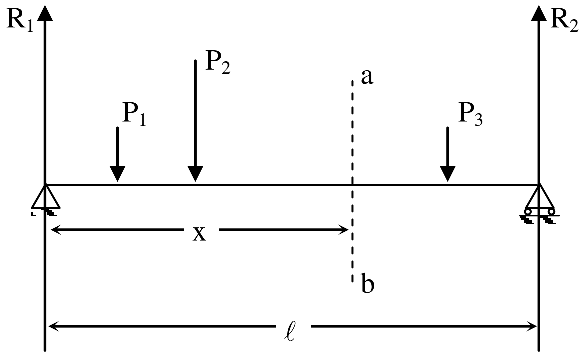
Similarly, by considering the moments with respect to the point A, we obtain,

$$R_2 = \frac{Pa}{\ell}$$

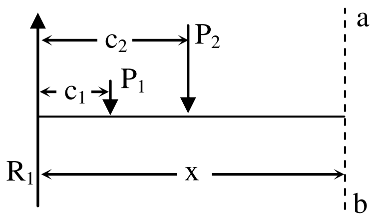
Bending Moment and Shearing Force :

Consider a simply supported beam acted upon by vertical forces P_1, P_2 and P_3 as shown below. Assume that the beam has an axial plane of symmetry and that the loads act in this plane. Then from symmetry, we conclude that bending must also occur in this same plane.

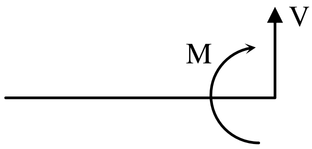
i) To investigate the stresses produced in a beam during bending, let us imagine that the beam AB is cut in two parts by a cross–section ab taken at any distance x from the left support, and that the portion of the beam to the right is removed.



(a)



(b)



(c)

In discussing the equilibrium of the remaining left hand portion of the beam, we consider not only the external forces such as loads P_1, P_2 and reaction R_1, but also the internal forces which are distributed over the cross-section ab and which represent the action of the right portion of the beam on the left portion. These internal forces must be of such a magnitude as to equilibrate the above mentioned external forces P_1, P_2 and R_1.

From statics, we know that a system of parallel forces can be replaced by one force equal to the algebraic sum of the given forces together with a couple. Thus, the balancing force here is,

$$V = R_1 - P_1 - P_2$$

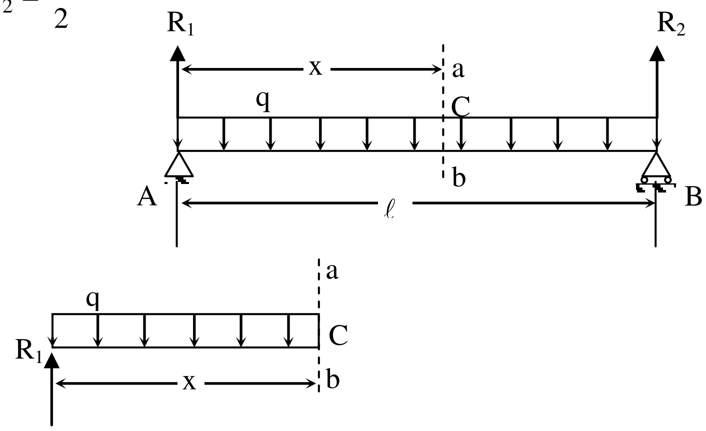
and the magnitude of the couple is

$$M = R_1 x - P_1\left(x - C_1\right) - P_2\left(x - C_2\right)$$

The force V, which is equal to the algebraic sum of the external forces to the left of the cross-section ab, is called the shearing force at the cross-section ab. The couple M, which is equal to the algebraic sum of the moments of the external forces to the left of the cross-section ab with respect to the centroid of this cross-section is called the bending moment at the cross-section ab.

ii) Let us now consider a distributed load rather than a number of concentrated forces acting on a beam, then denoting the load per unit length by q, the reactions in this case are,

$$R_1 = R_2 = \frac{q\ell}{2}$$



Let us investigate the stresses distributed over a cross-section ab by considering the equilibrium of the left portion of the beam, as shown in the above figure. The external forces acting on this portion of the beam are the reaction R_1 and the load uniformly distributed along the length x. This latter load has a resultant equal to qx. The algebraic sum of all forces to the left of the cross-section ab is $R_1 - qx$. The moment of the distributed load about point C is evidently equal to,

$$qx \times \frac{x}{2} = \frac{qx^2}{2}$$

We obtain for the algebraic sum of the moments the expression,

$$R_1 x - \frac{qx^2}{2}$$

All the forces acting on the left portion of the beam can now be replaced by one force acting in the plane of the cross-section ab and equal to

$$V = R_1 - qx = q\left[\frac{\ell}{2} - x\right]$$

together with a couple equal to,

$$M = R_1 x - \frac{qx^2}{2} = \frac{qx}{2}(\ell - x)$$

Sign Conventions :

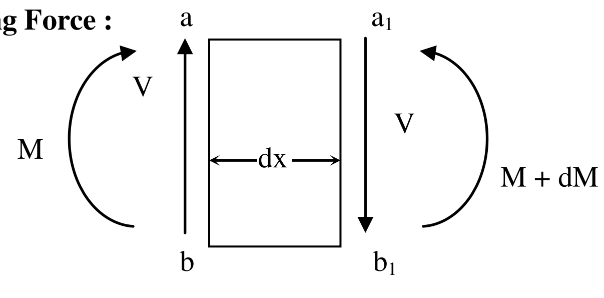
The bending moment and the shearing force at a cross-section have the following sign conventions.

Bending Moment

Shearing Force

Relation between Bending Moment and Shearing Force :

Consider an element of a beam cut out by two adjacent cross-sections ab and a_1b_1 which are at a distance shearing force at the cross-section ab, the action of the left portion of the beam on the element is represent by the force V and the couple M as shown in the figure below.



If no forces act on the beam between cross-sections ab and a_1b_1 the shearing forces at these two sections are equal. Regarding the bending moment, there is an increase which equals the moment of the couple represented by the two equal and opposite forces V, i.e. ,

$$dM = Vdx$$

and

$$\frac{dM}{dx} = V$$

Similarly, for a distributed load of intensity q acting between the cross-sections ab and a_1b_1, the total load acting on the element is q dx.

Then,

$$dV = -q\,dx$$

which gives,

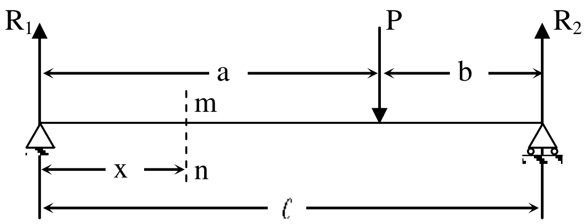
$$\frac{dV}{dx} = -q$$

Bending Moment and Shear Force Diagrams :

A representation in which the abscissa indicate the position of the cross-section and the ordinate represents the value of the bending moment or shearing force which acts at this cross-section, is called Bending Moment and Shearing Force Diagrams.

i) Consider a simply supported beam with a single concentrated load P.
The reactions in this case are,

$$R_1 = \frac{Pb}{\ell} \text{ and } R_2 = \frac{Pa}{\ell}$$



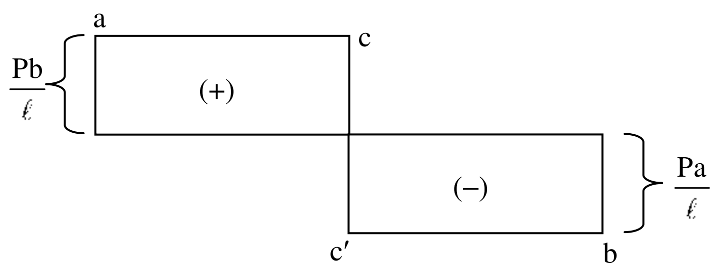
Taking a cross-section mn to the left of P, it can be concluded that at such a cross-section,

$$V = \frac{Pb}{\ell} \text{ and } M = \frac{Pb}{\ell}x$$

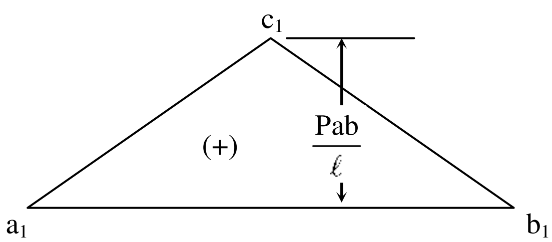
For a cross-section to the right of the load, we obtain

$$V = \frac{Pb}{\ell} - P \quad \text{and} \quad M = \frac{Pb}{\ell}x - P(x-a),$$

x always being the distance from the left end of the beam.



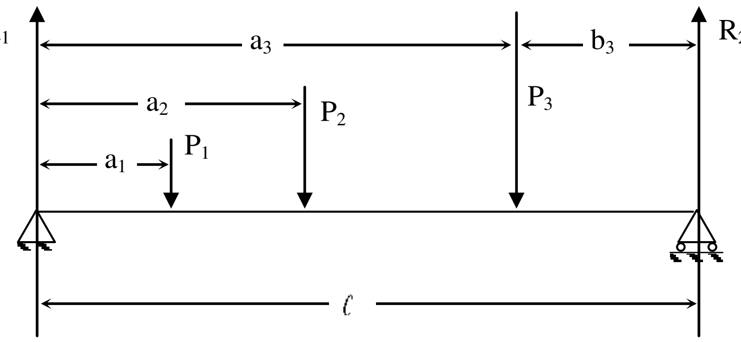
Shear Force Diagram

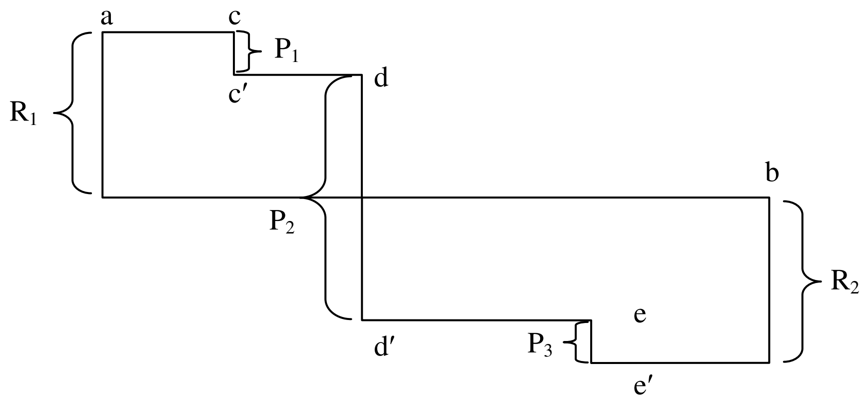


Bending Moment Diagram

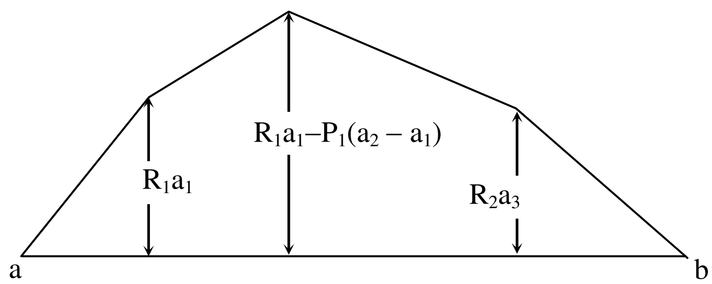
The shearing force and bending moment diagrams are drawn as shown above.
Note that the shearing force diagram consists of two rectangles of equal areas. Also, the rectangles are in opposite directions.
In the case of a simply supported beam, the moments at the end vanish.

ii) If several loads act on a beam, the beam is divided into several portions and expressions for V and M are established for each portion, as shown below.



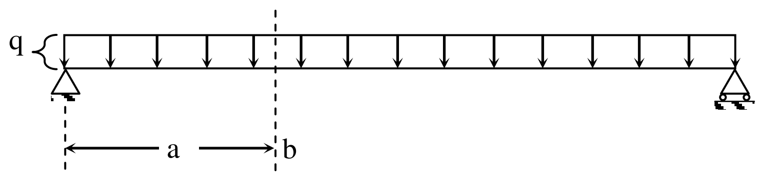


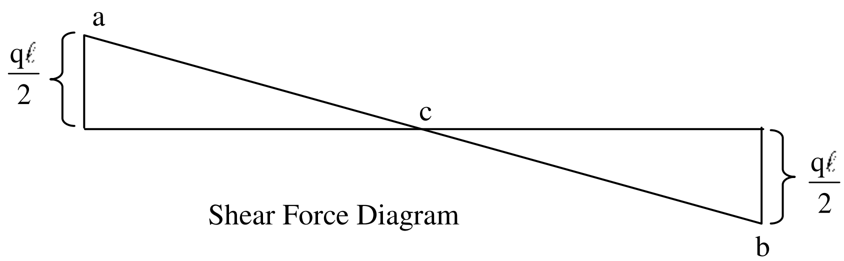
Shear Force Diagram



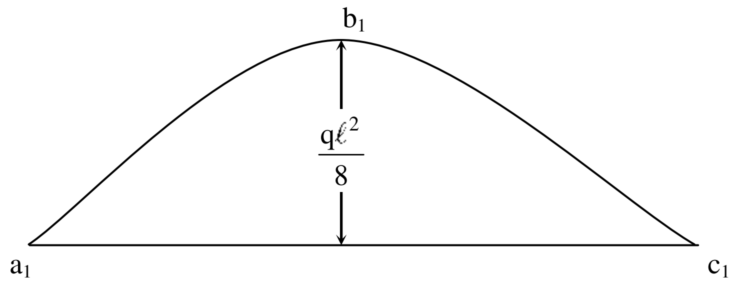
Bending Moment Diagram

iii) Consider the case of a uniformly distributed load, as shown below :





Shear Force Diagram



Bending Moment Diagram

Now at the cross-section ab at a distance x from the left support,

$$V = q\left(\frac{\ell}{2} - x\right) \quad \text{and}$$

$$M = \frac{qx}{2}(\ell - x)$$

We see here that the shearing force diagram consists of an inclined straight line. Also, the bending moment here is a parabolic curve with its vertical axis at the middle of the span of the beam. The maximum value is obtained at the middle of the span, giving,

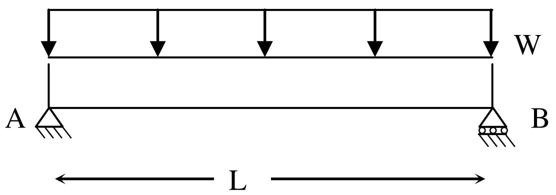
$$M_{max} = \frac{q\ell^2}{8}$$

If concentrated loads and distributed loads act on the beam simultaneously, it is advantageous to draw the diagrams separately for each kind of loading and obtain the total values of V or M at any cross-section by summing up the corresponding ordinates of the two partial diagrams.

Example :

A simply supported beam of length L, cross-section A, carrying a uniformly distributed load of W will have maximum bending moment of:

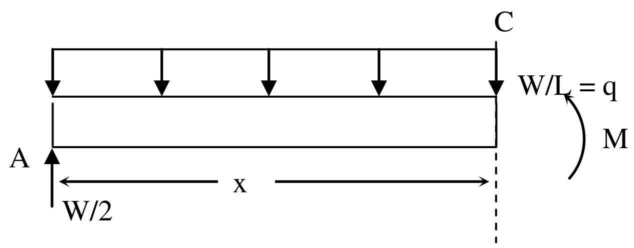
Solution :



Due to symmetry, the maximum bending moment will occur at the center. Now, taking a cross section at the center, and doing a moment balance about point A, we have,

M = (W/L x L/2) x L/4 = WL/8, where the first term is the net force due to the distributed load and the second term is the distance from A at which it is acting.

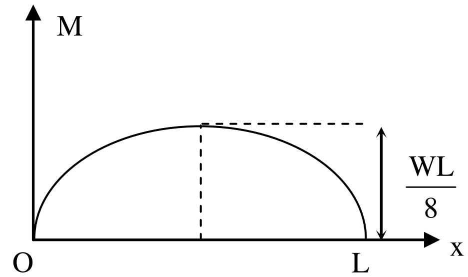
The bending moment at any distance x from point A is :



Taking moment about C, we get,

$$M = \left(\frac{W}{2}x\right) - \left(\frac{W}{L}x\right)\frac{x}{2} = \frac{W}{2}x - \frac{Wx^2}{2L}, \qquad 0 < x < \frac{L}{2}$$

$\therefore$ Bending moment diagram looks like :



The relationship between M and $\frac{W}{L} = q$ is as follows :

$$M = \frac{W}{2}x - \frac{Wx^2}{2L} = \frac{qL}{2}x - \frac{qx^2}{2}$$

$\therefore$ $\frac{dM}{dx} = -qx + \frac{qL}{2}$ = shear stress at any section x.

& $\frac{d^2M}{dx^2} = -q + 0$

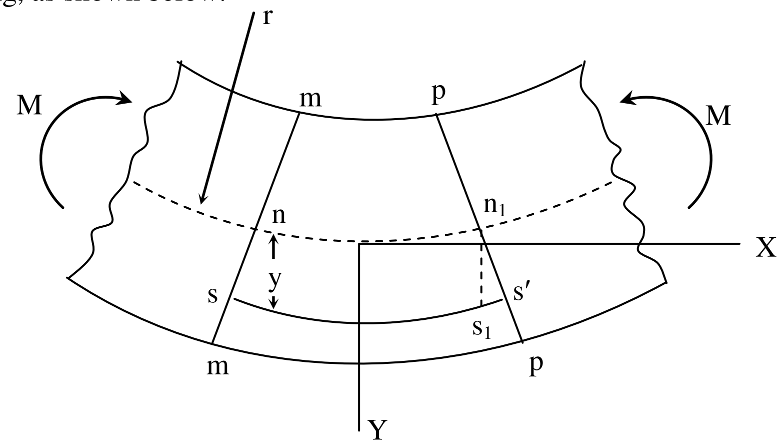
Thus distributed load q is second differential of moment M.

BENDING STRESS

Bending & Shear Stress :

Pure Bending : The magnitude of the stresses at any cross-section of a beam is defined by the magnitude of the shearing force and bending moment at the cross-section. The case where the shearing force is zero and only bending moment is present is called Pure Bending.

If the beam is of rectangular cross-section and two adjacent vertical lines mm and pp are drawn on its sides, the cross-sections mn and pp rotate with respect to each other about axes perpendicular to the plane of bending, as shown below.



The line nn_1 is the trace of the surface in which the fibers do not undergo strain during bending. This surface is called as neutral surface and the axis is called as neutral axis.

Denoting by r the radius of curvature of the deflected axis of the beam and using the similarity condition, the unit elongation of the fiber ss′ is

$$\varepsilon_x = \frac{s's_1}{nn_1} = \frac{y}{r}$$

The unit strain in the lateral direction is,

$$\varepsilon_2 = -\mu\varepsilon_x = -\mu\frac{y}{r}$$ where μ is Poisson's Ratio.

The radius of curvature R, of all straight lines in the cross-section, will be larger than r in the same proportion in which ε_x is numerically larger than ε_z and we obtain,

$$R = \frac{1}{\mu} r$$

Using Hook's Law and the strain condition we obtained earlier, we get

$$\sigma_x = \frac{Ey}{r}$$

The force acting on any element dA at distance y from the neutral axis is given by $\left(\frac{Ey}{r}\right) dA$. The resultant of these forces in the x – direction must be equal to zero and we obtain,

$$\int \frac{Ey}{r} dA = \frac{E}{r} \int y dA = 0$$

i.e. the moment of the area of the cross-section with respect to the neutral axis is equal to zero. Hence the neutral axis passes through the centroid of the section.

The moment of the force acting on the element dA with respect to the neutral axis is $\left(\frac{Ey}{r}\right) dA \cdot y$.

Adding all such moments we get

$$\int \frac{E}{r} y^2 dA = \frac{EI_z}{r} = M$$

$$\Rightarrow \quad \frac{1}{r} = \frac{M}{EI_z}$$

where,

$$I_z = \int y^2 dA$$

is the moment of inertia of the cross-section with respect to the neutral axis z.

Using all the above derived equations,
we can eliminate r, and

$$\sigma_x = \frac{My}{I_z}$$

We develop a relationship for the bending stress which develops in a loaded beam. This relationship is known as the **Flexure Formula**. In the diagram below we have shown a simply supported beam loaded at the center. It deflects (or bends) under the load.

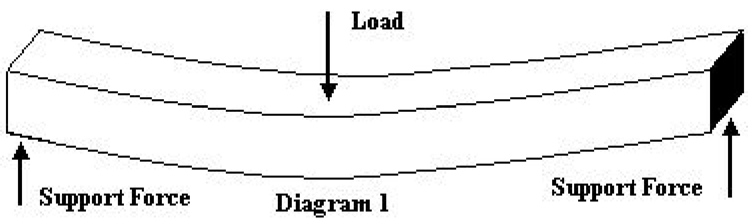
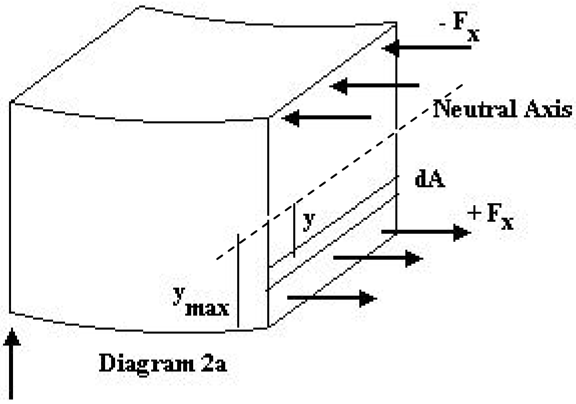


Diagram 1

In the lower diagram, we have shown the left end section of the beam. As discussed previously, when examining bending moments, horizontal forces act on the cross-sectional face of the beam section. We have shown only the horizontal forces along the top and bottom in Diagram 2a, but the forces act across the whole cross-section as shown in the side view in Diagram 2b. The horizontal forces decrease from maximum at the outer edges to zero at the neutral axis (an axis running through the centroid of beam cross-section).



Neutral Axis

This is the axis along the length of the beam which remains unstressed, neither compressed nor stretched when it is bent. Normally the neutral axis passes through the centroid of the cross-sectional area. The position of the centroid is hence important. Consider that the beam is bent into an arc of a circle through angle θ radians. AB is on the neutral axis and is the same length before and after bending. The radius of the neutral axis is R.

Radius of Curvature

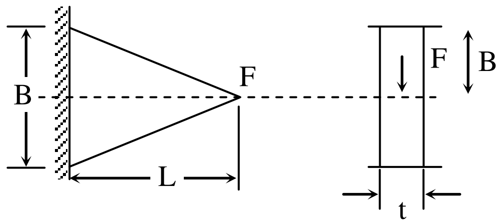
Normally the beam does not bend into a circular arc. However, what ever shape the beam takes under the sideways loads; it will basically form a curve on an x – y graph. In maths, the radius of curvature at any point on a graph is the radius of a circle that just touches the graph and has the same tangent at that point.

Flexure Formula

It is given by

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

The beam can be designed such that stress at every section in the beam is same. Such a beam is called uniform strength beam.



Uniform Strength Beam

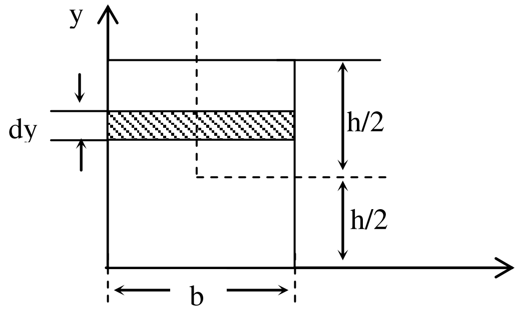
where B = width at fixed end
L = Length from fixed end
t = thickness of beam

MOMENT OF INERTIA OF PLANE AREAS

Moment of Inertia of a plane area w.r.t. an axis in its plane – The moment of inertia of the area A with respect to the z – axis is given by

$$I_z = \int_A y^2 dA$$

For example, the moment of inertia of a rectangle w.r.t. the horizontal axis of symmetry z is calculated as,



$$I_z = 2\int_0^{h/2} y^2 b dy = \frac{bh^3}{12}$$

Similarly, the moment of inertia of a rectangle w.r.t. the y – axis is,

$$I_y = 2\int_0^{b/2} z^2 h dz = hb^3/12$$

POLAR MOMENT OF INERTIA OF A PLANE AREA

The moment of inertia of a plane area w.r.t. an axis perpendicular to the plane of the figure is called the polar moment of inertia w.r.t. the point where the axis intersects the plane.
It is defined as

$$I_p = \int_A r^2 dA$$

Thus for a circular cross-section we take $dA = 2\pi r dr$ and

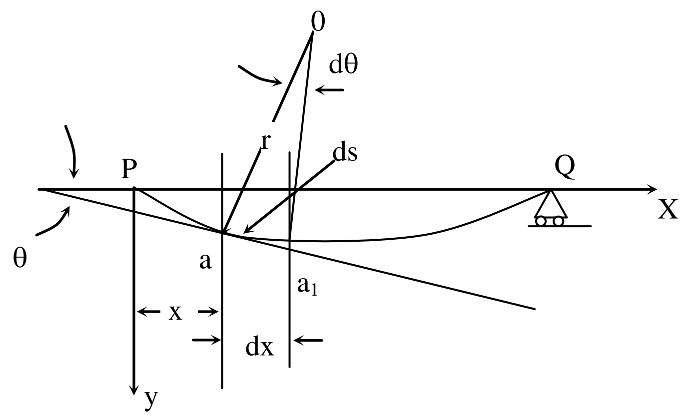
$$I_p = 2\pi\int_0^{d/2} r^3 dr = \pi d^4/32$$

Also $r^2 = y^2 + z^2$

$$\therefore \quad I_p = \int_A r^2 dA = \int_A \left(y^2 + z^2\right) dA = I_y + I_z$$

DEFLECTION OF BEAMS

Differential Equation of the deflection curve – Normally, one is interested not only in the stresses produced by the acting loads but also in the deflections produced by these loads.



Let the curve PaQ in the above figure represent the shape of the axis of the beam after bending. Bending takes place in the plane of symmetry due to transverse forces acting in that plane. This curve is called the deflection curve. To derive the differential equation of this curve, we take the coordinate axes as shown in the figure and assume that the curvature of the deflection curve at any point depends only on the magnitude of the bending moment M at that point. In such a case, the relation between the curvature and the bending moment is the same as in the case of pure bending and we obtain,

$$\frac{1}{r} = \frac{M}{EI_Z}$$

Let us consider two adjacent points a and a_1, distance ds apart on the deflection curve. If the angle which the tangent at a makes with the x − axis is denoted by θ the angle between the normals to the curve at a and a_1 is dθ. The intersection point of these normals gives the centre of curvature and defines the length r of the radius of curvature.

Then,

$$ds = r d\theta$$

and

$$\frac{1}{r} = \left|\frac{d\theta}{ds}\right|$$

In practical applications only very small deflections of beams are allowable and the deflection curves are very flat. In such cases we can assume with sufficient accuracy that,

$ds \approx dx$ and $\theta \approx \tan\theta = dy/dx$.

Substituting these values, we obtain,

$$\frac{1}{r} = -\frac{d^2y}{dx^2}$$

Thus, we finally get,

$$EI_z \frac{d^2y}{dx^2} = -M \qquad \ldots(1)$$

In the case of very long slender bars, in which the deflection may be large, it is not permissible to use the simplifications of θ used previously, and we use the exact expression,

$$\theta = \text{arc tan}\left(\frac{dy}{dx}\right) \qquad \ldots(2)$$

Then,

$$\frac{1}{r} = -\frac{d\theta}{ds} = -\frac{d\left[\text{arc tan}\left(\frac{dy}{dx}\right)\right]}{dx}.\frac{dx}{ds}$$

$$= -\frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}} \qquad \ldots(3)$$

By differentiating equation (1) w.r.t. x we obtain

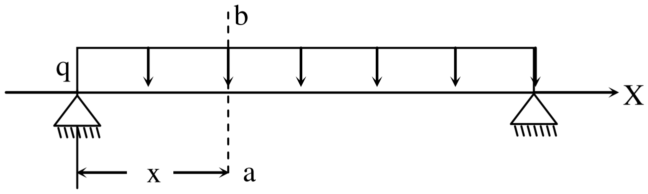
$$EI_z \frac{d^3y}{dx^3} = -V \qquad \ldots(4)$$

and

$$EI_z \frac{d^4y}{dx^4} = q \qquad \ldots(5)$$

Bending of a uniformly Loaded Beam :

In the case of a simply supported and uniformly loaded beam,



The bending moment at any cross-section ab, a distance x from the left support is,

$$M = \frac{q\ell x}{2} - \frac{qx^2}{2} \qquad \ldots(6)$$

and the differential equation becomes,

$$EI_z \frac{d^2y}{dx^2} = -\frac{q\ell x}{2} + \frac{qx^2}{2} \qquad \ldots(7)$$

Multiplying both sides by dx and integrating we obtain

$$EI_Z \frac{dy}{dx} = -\frac{q\ell x^2}{4} + \frac{qx^3}{6} + C \qquad \ldots(8)$$

where C is the constant of integration, which is obtained as a result of symmetry which gives the slope at the middle of the span as zero. Setting dy/dx = 0 when x = $\ell/2$, we obtain,

$$C = \frac{q\ell^3}{24}$$

Thus,

$$EI_Z \frac{dy}{dx} = -\frac{q\ell x^2}{4} + \frac{qx^3}{6} + \frac{q\ell^3}{24} \qquad \ldots(9)$$

A second integration gives,

$$EI_Z\, y = -\frac{q\ell x^3}{12} + \frac{qx^4}{24} + \frac{q\ell^3 x}{24} + C_1 \qquad \ldots(10)$$

The new constant of integration C_1 is determined from the condition that the deflections at the supports is zero. Substituting y = 0 and x = 0 in the above equation, we get $C_1 = 0$

Thus,

$$y = \frac{q}{24EI_z}\left(\ell^3 x - 2\ell x^3 + x^4\right)$$

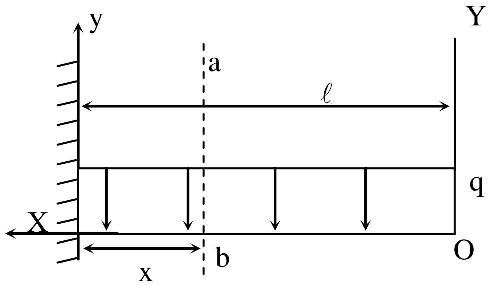
This is the deflection curve of a simply supported and uniformly loaded beam. The maximum deflection of this beam is evidently at the middle of the span. Substituting x = ℓ /2, we get

$$y_{max} = \frac{5}{384} \cdot \frac{q\ell^4}{EI_z}$$

The maximum slope occurs at the left end of the beam where by substituting x = 0 in equation(10) we obtain,

$$\left(\frac{dy}{dx}\right)_{max} = \frac{q\ell^3}{24EI_z}$$

In the case of a uniformly loaded cantilever beam,



The bending moment at a cross-section ab a distance x from the left end is

$$M = -\frac{qx^2}{2}$$

and equation(1) becomes,

$$EI_Z \frac{d^2y}{dx^2} = \frac{qx^2}{2}$$

Integrating once we get,

$$EI_Z \frac{dy}{dx} = \frac{qx^3}{6} + C$$

C, the constant of integration is found from the condition that the slope at the built in end is zero, that is dy/dx = 0 for x = ℓ. Substituting these values in the above equation, we get,

$$C = -\frac{-q\ell^3}{6}$$

The second integration gives

$$EI_z y = \frac{qx^4}{24} - \frac{q\ell^3 x}{6} + C_1$$

The constant C_1 is found from the condition that the deflection vanishes at the built end. Thus, by substituting x = ℓ, y = 0, in the above equation, we get

$$C_1 = \frac{q\ell^4}{8}$$

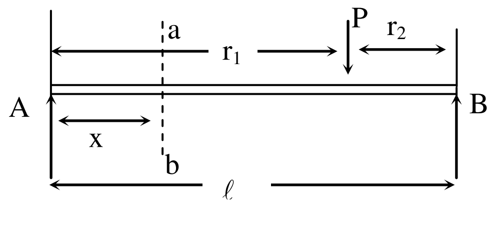
Substituting this value, we get

$$y = \frac{q}{24EI_Z}\left[x^4 - 4\ell^3 x + 3\ell^4\right]$$

This equation defines the deflection curve of the uniformly loaded cantilever.

Deflection of a simply supported beam loaded by a concentrated load

In this case there are two different expressions for the bending moment corresponding to the two portions of the beam.



The equation,

$$EI_Z \frac{d^2y}{dx^2} = -M$$

For the deflection curve must therefore be written for each portion as,

$$EI_Z \frac{d^2y}{dx^2} = -\frac{Pbx}{\ell} \quad \text{for} \quad x \le a,$$

and

$$EI_Z \frac{d^2y}{dx^2} = -\frac{Pbx}{\ell} + P(x-a) \quad \text{for} \;\; x \ge a.$$

By integrating these equations we obtain,

$$EI_Z \frac{dy}{dx} = \frac{-Pbx^2}{2\ell} + C, \;\; \text{for} \quad x \le a$$

and

$$EI_Z \frac{dy}{dx} = \frac{-Pbx^2}{2\ell} + \frac{P(x-a)^2}{2} + C_1, \quad \text{for} \quad x \ge a.$$

Since the two branches of the deflection curve must have a common tangent at the point of application of the load P, the above expressions for the slope must be equal for x = a. Thus, we can conclude that $C = C_1$. Performing the second integration, we get,

$$EI_Z\, y = -\frac{Pbx^3}{6\ell} + Cx + C_2 \;\; \text{for} \quad x \le a$$

and

$$EI_Z\, y = \frac{-Pbx^3}{6\ell} + \frac{P(x-a)^3}{6} + Cx + C_3 \;\; \text{for} \;\; x \ge a.$$

Again, since the two branches of the deflection curve have a common deflection at the point of application of the load, the above expressions must be identical for x = a. Therefore, $C_2 = C_3$.

Substituting x = 0 and y = 0 in the above expression for x ≤ a, we get,

$$C_2 = C_3 = 0$$

Substituting y = 0 and x = ℓ in the above expression for x ≥ 0, we get,

$$C = \frac{Pb\ell}{6} - \frac{Pb^3}{6\ell} = \frac{Pb\left(\ell^2 - b^2\right)}{6\ell}$$

Finally substituting the above constant values, we obtain,

$$EI_Z\, y = \frac{Pbx}{6\ell}\left(\ell^2 - b^2 - x^2\right) \;\; \text{for} \quad x \le a.$$

$$EI_Z y = \frac{Pbx}{6\ell}\left(\ell^2 - b^2 - x^2\right) + \frac{P(x-a)^3}{6} \quad \text{for } x \ge a$$

The first equation gives the expression for deflection for the left portion of the beam, and the second equation gives the deflection for the right portion.

The slope at any point can be obtained by,

$$EI_Z \frac{dy}{dx} = \frac{Pb}{6\ell}\left(\ell^2 - b^2 - 3x^2\right) \;\; \text{for} \quad x \le a.$$

$$EI_Z \frac{dy}{dx} = \frac{Pb}{6\ell}\left(\ell^2 + b^2 + 3x^2\right) + \frac{P(x-a)^2}{2} \;\; \text{for} \quad x \ge a.$$

We get,

$$\left.\frac{dy}{dx}\right|_{x=0} = \frac{Pb\left(\ell^2 - b^2\right)}{6\ell EI_Z}$$

and

$$\left.\frac{dy}{dx}\right|_{x=\ell} = -\frac{Pab(\ell + a)}{6\ell EI_Z}$$

Also

$$y_{max} = \left. y\right|_{x=\ell/2} = \frac{P\ell^3}{48EI_Z}$$

Deflection by Use of Bending Moment Diagram – [Area Moment Method]:

The deflection at a prescribed point can be calculated considerably simplified by the use of bending moment diagram, rather than the general equation of the deflection curve.

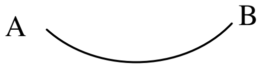


Fig: Moment Applied on beam

$$\delta = \int_A^B \frac{1}{EI_z} xM\,dx$$

$$\theta = \int_A^B \frac{1}{EI_Z} Mdx$$

The above equation states that the distance of B from the tangent at A is equal to the moment w.r.t. the vertical through B of the area of the bending moment diagram between A and B, divided by the flexural rigidity EI_Z.

Using the above equations, the slope of the deflection curve and the magnitude of deflection curve and the magnitude of deflection at any cross-section of the beam can easily be calculated in each particular case.

We first calculate the absolute values of θ and δ. Then taking the positive directions of the coordinate axes, we consider the rotation of a tangent to the deflection curve as positive if it is in the clockwise direction, and the deflection of the beam as positive if it is in the direction of the positive y – axis. This method of calculating deflections is called the area moment method.

Deflection of a cantilever beam by the area moment method:

For the case of a cantilever beam with a concentrated load at the end, the bending moment diagram is shown below.

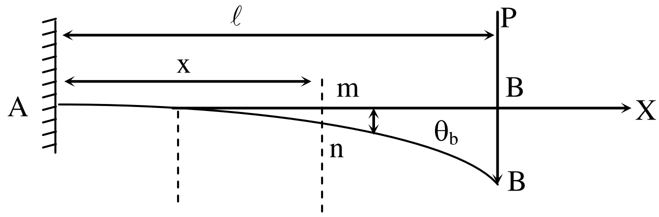


Fig. Cantilever Beam

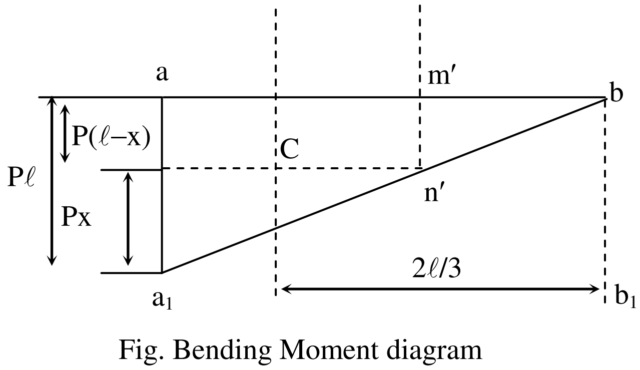


Fig. Bending Moment diagram

Angle θ_b which the tangent to the deflection curve at B makes with the tangent at A is,

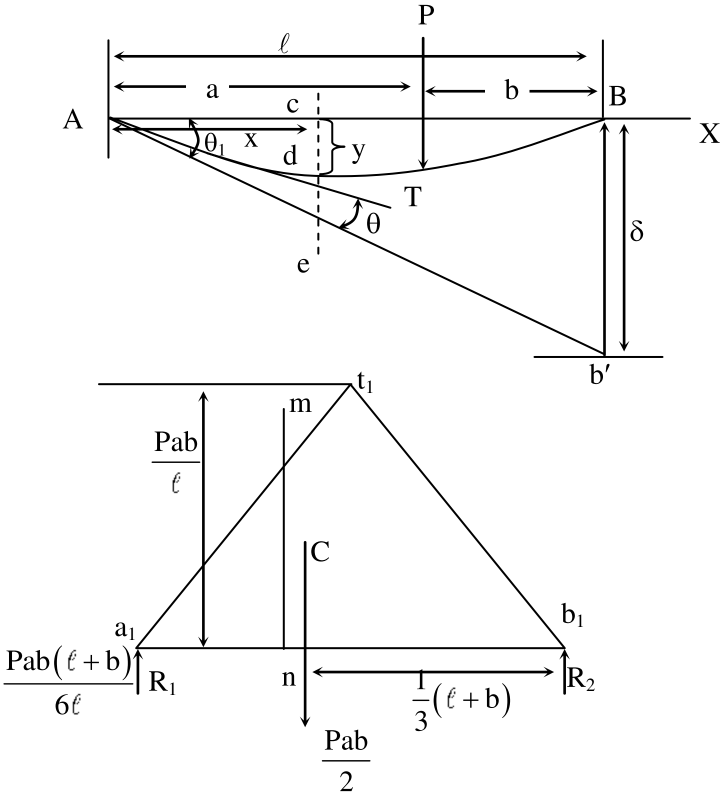
$$\theta_b = P\ell \times \frac{\ell}{2} \times \frac{1}{EI_Z} = \frac{P\ell^2}{2EI_Z}$$

The deflection δ is calculated as the moment of the area aba_1 about the axis bb_1 divided by EI_z. Thus,

$$\delta = P\ell \times \frac{\ell}{2} \times \frac{2\ell}{3} \times \frac{1}{EI_z} = \frac{P\ell^3}{3EI_z}$$

Deflection of a simply supported beam by the moment area method.

Let us consider a simply supported beam with a load P at point T.



The bending moment diagram is the triangle $a_1b_1t_1$. Its area is $\frac{Pab}{2}$, and its centroid c is at distance $\frac{(\ell + b)}{3}$ from the vertical Bb_1. The vertical distance δ from the end B to the line Ab′ which is tangent to the deflection curve at A is obtained as,

$$\delta = \frac{1}{EI_Z}.\frac{Pab}{2} \times \left(\frac{\ell + b}{3}\right) = \frac{Pab(\ell + b)}{6EI_Z}$$

By using this value, the slope θ_1 at the left end of the beam is found to be,

$$\theta_1 = \frac{\delta}{\ell} = \frac{Pab(\ell + b)}{6\ell EI_2}$$

Also, we obtain,

$$R_1 = \frac{Pab(\ell + b)}{6\ell}$$

and

$$R_2 = \frac{Pab(\ell + a)}{6\ell}$$

Effect of shearing force on the deflection of beams :

Shearing force causes an additional deflection along with that produced by the bending moment, due to the mutual sliding of adjacent cross-sections along each other. As a result of the non–uniform distribution of the shearing stresses, the cross-sections become curved, which shows the bending due to shear alone.

Denoting by y_1 the deflections due to shear, we obtain for any cross–section the following expression for the slope :

$$\frac{dy_1}{dx} = \frac{(\tau_{xy})_{y=0}}{G} = \frac{\alpha V}{AG}$$

in which V/A is the average shearing stress τ_{xy}, G is the modulus of shear and α is a numerical factor. For a rectangular cross-section $\alpha = \frac{3}{2}$; for a circular cross-section $\alpha = \frac{4}{3}$.

Using this formula, and depending on the specific conditions of the given problem, one can integrate the above formula to obtain the value of y_1.

When a beam bends it takes up various shapes. The shapes may be superimposed on x – y graph with the origin at the left end of the beam (before it is loaded). At any distance x metres from the left end, the beam will have a deflection y and a gradient or slope dy/dx and it is these that we are concerned with. For a distributed load of q_0 per unit length deflection,

$$y = \frac{q_0 x}{24EI}\left[L^3 - 2Lx^2 + x^3\right] \text{ at any section x.}$$

For a cantilever beam with load W at free end, the deflection is

$$y_{max} = \frac{WL^3}{2EI} \text{ and bending moment is}$$

$$B.M_{max} = W.L$$

The deflection caused by moment M applied at free end is

$$y = \frac{ML^2}{2EI}$$

The equation relating bending moment and radius of curvature in a beam is :

$$\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}$$

$$\Rightarrow \quad R = \frac{Ey}{\sigma}$$

M is the bending moment.
I is the second moment of area about the centroid.
E is the modulus of elasticity and
R is the radius of curvature.

Mathematically, it can be shown that any curve plotted on x – y graph has a radius of curvature of the beam R, can be defined as

$$\frac{1}{R} = \frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}} = \frac{1}{R} = \frac{y''}{\left[1+(y')^2\right]^{3/2}}$$

In beams, R is very large and the equation may be simplified without loss of accuracy to:

$$\frac{1}{R} = \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

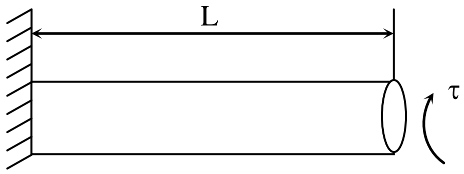
$$M = EI\frac{d^2y}{dx^2}$$

The product EI is called the flexural stiffness of the beam.

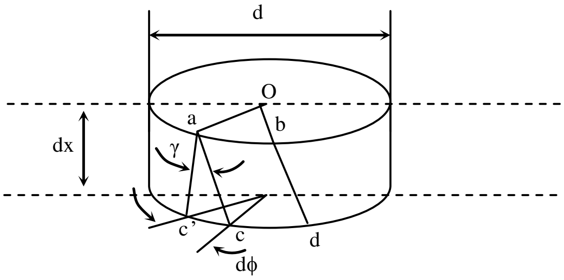
THE THEORY OF SUPERPOSITION FOR COMBINED LOADS

This theory states that the slope and deflection of a beam at any point is the sum of the slopes and deflections which would be produced by each load acting on its own. For beams with combinations of loads which are standard cases we only need to use the standard formulae.

TORSION OF CIRCULAR SHAFTS



When a circular shaft supported at the upper end is twisted by a couple at the lower end, the shaft rotates at the lower end of each cross-section taken with respect to the top cross-section through an angle $d\phi$.



The rectangular element abcd becomes disoriented as shown in the figure above. The element is in a state of pure shear and the magnitude of the shearing strain γ is,

$$\gamma = \frac{c'c}{ac'}$$

Now, $c'c$ is the small arc of radius $(d/2)$ corresponding to the difference $d\phi$ in the angle of rotation of the two adjacent cross-sections, $c'c = (d/2)\, d\phi$, and we get,

$$\gamma = \frac{1}{2}\left(\frac{d\phi}{dx}\right)d$$

For a shaft twisted by a torque at the end the angle of twist is proportional to the length and the quantity $d\phi/dx$ is constant. It represents the angle of twist per unit length of the shaft and is called θ. then

$$\gamma = \frac{1}{2}\theta d$$

The shearing stress which acts on the sides and produces the above shear is given by,

$$\tau = \frac{1}{2}G\,\theta d$$

Here, an assumption is made that only the circular boundaries of the cross-sections of the shaft remains undistorted and the cross-section remains plane and rotates as if absolutely rigid, i.e. every diameter of the cross-section remains straight and rotates the same angle.

We see that the shearing stress varies directly as the distance r from the axis of the shaft. The maximum stress occurs in the surface layer of the shaft. The relationship between the applied twisting couple M_t and the stresses it produces is now developed. The shearing stresses distributed over the cross-section are statically equivalent to a couple equal and apposite to the torque M_t. For each element having area dA, the shearing force is τ dA. The moment of this force about the axis of the shaft is $(\tau dA)\, r = G\theta r^2 dA$.

The torque M_t is the summation taken over the entire cross-sectional area of these moments i.e.

$$M_t = \int_A G\theta r^2 dA = G\theta \int_A r^2 dA = G\theta I_P$$

where I_p is the polar moment of inertia of the circular cross-section. For a circle of diameter d we have,

$$I_p = \frac{\pi d^4}{32}$$

and therefore

$$M_t = G\theta \cdot \frac{\pi d^4}{32}$$

$$\Rightarrow \qquad \theta = \frac{M_t}{G}\frac{32}{\pi d^4} = \frac{M_t}{GI_P}$$

It can be seen that θ, the angle of twist per unit length of the shaft, varies directly as the applied torque and inversely at the modular of shear G and the fourth power of the diameter.
The total angle of twist of a shaft of length ' ' is,

$$\phi = \theta \quad = \frac{M_t \ell}{GI_p}$$

From the above equations, we can calculate maximum shearing stress in torsion for a circular shaft,

$$\tau_{max} = \frac{M_t d}{2I_p} = \frac{16M_t}{\pi d^3}$$

Torsion of a hollow shaft $\rightarrow$ For a solid shaft, the material at the outer surface of the shaft will be stressed to the limit assigned as the working stress.
In calculating the moment of the shearing stresses for a hollow shaft, the radius r varies from the radius of the inner hole, denoted by $\frac{1}{2}d_1$, to the outer radius of the inner hole, denoted by $\frac{1}{2}d$. Then

$$G\theta \int_{\frac{1}{2}d_1}^{\frac{1}{2}d} r^2 dA = M_t = G\theta I_p$$

where $I_p = (\pi/32)\,(d^4 - d_1^4)$ is the polar moment of inertia of the ring section. Then,

$$\theta \quad = \frac{32M_t}{\pi\left(d^4 - d_1^4\right)G} = \frac{M_t}{GI_p}$$

and the angle of twist will be

$$\phi = \theta\ell = \frac{M_t \ell}{GI_P}$$

Finally,

$$\tau_{max} = \frac{16M_t}{\pi d^3\left(1 - \frac{d_1^4}{d^4}\right)} = \frac{M_t d}{2I_{p.}}$$

In engineering solid mechanics, in analyzing members for torque, regardless of the type of cross-section, the basic method of sections is employed. For the torsion problems, the equation used is:

$$\Sigma M = 0$$

Therefore, for statically determinate systems, there can only be one reactive torque. After determining this torque, we take sections to compute the internal torque, which must balance the externally applied torques. The resultant torque about the axis of a circular shaft due to a shear stress distribution τ(r), an arbitrary function of *r* is,

$$M_T = 2\pi \int_0^R \tau(r) r^2 dr$$

Now, with our linear function of *r*, we can carry out the integration. Doing so we obtain a relationship between the applied torque M_T and the **rate of rotation** $\frac{d\phi}{dz}$, a *stiffness* relation.

$$M_T = 2\pi \int_0^R G \frac{d\phi}{dz} r^3 dr$$

or, since $\frac{d\phi}{dz}$ and *G* are constants, we are left with the integral of r^3 and can write

$$M_T = GJ\left(\frac{d\phi}{dz}\right)$$

where *J* is a function only of the geometry of the cross-section - its radius *R*. You may have encountered it as the *polar moment of inertia*

$$J = \int_{Area} r^2 dA$$

For the circular shaft,

$$J = \pi \frac{R^4}{2}$$

This stiffness equation is analogous to the stiffness relationship derived for one bay of the truss structure considered at the outset of the chapter. For a shaft of length *L*, the rotation of one end relative to the other is just the integral of the **constant** rate of rotation over the length, that is, just the product of the two. We obtain then:

$$M_T = \left(\frac{GJ}{L}\right) \cdot \phi$$

We can obtain the shear stress and strain distribution in terms of the applied moment by substitution.

We obtain

$$\tau(r) = r \cdot \left(\frac{M_T}{J}\right)$$

$$\gamma(r) = r \cdot \frac{M_T}{(GJ)}$$

Our analysis is identical for a hollow shaft. All of the symmetry arguments apply. Only the expression for J changes: It becomes

$$J = \int_{Area} r dA = \left(\frac{\pi}{2}\right)\left(R_o^4 - R_i^4\right)$$

Design for circular members in Torsion for Strength

For a shaft rotating with a frequency of f Hz, the angle is 2πf rad/s. Hence, if a shaft were transmitting a constant torque T measured in N.m, it would do 2πfT Nm of work/sec.

Equating this to the horsepower supplied,

$\text{Hp} \times 745.7 = 2\pi fT$ Nm/s

or, $T = 119 \times \text{hp}/f$ (Nm)

or, $T = 159 \times \text{KW}/f$ (Nm)

Since $\frac{2\pi fT}{1000} = \text{PKW}$

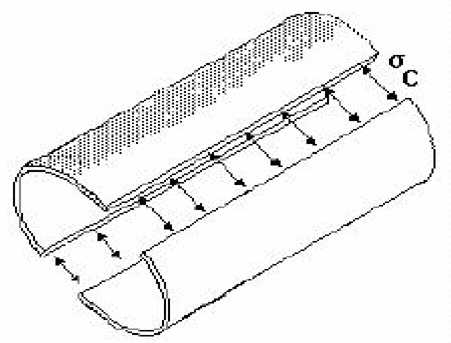
$\therefore \quad T = \frac{1000}{2\pi f}P = \frac{159}{f}P$ where P is in KW.

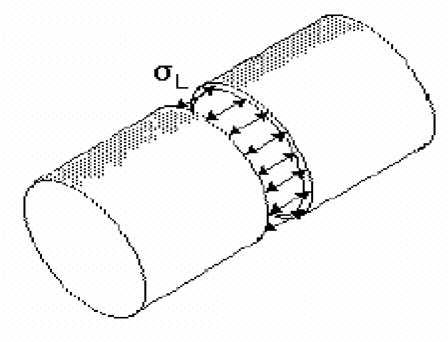
THIN CYLINDRICAL AND SPHERICAL SHELLS

THIN WALLED CYLINDER

A cylinder is regarded as thin walled when the wall thickness t is less than $1/20^{th}$ of the diameter D. When the wall is thicker than this, it is regarded as a thick wall and it is treated differently.

Consider a cylinder of mean diameter D, wall thickness t and length L. When the pressure inside is larger than the pressure outside by p, the cylinder will tend to split along a length and along a circumference as shown in figures below.





The stress produced in the longitudinal direction is σ_L and in the circumferential direction is σ_C. These are called the longitudinal and circumferential stresses respectively. The latter is also called the hoop stress.

Consider the forces trying to split the cylinder about a circumference in the figure above (right side). So long as the wall thickness is small compared to the diameter then the force trying to split it due to the pressure is

$$F = pA = p\frac{\pi D^2}{4} \qquad \ldots(1.1)$$

So long as the material holds then the force is balanced by the stress in the wall. The force due to the stress is

$F = \sigma_L$ multiplied by the area of the metal $= \sigma_L \pi Dt$ …(1.2)

Equating 1.1 and 1.2 we have,

$$\sigma_L = \frac{pD}{4t} \qquad \ldots(1.3)$$

Now consider the forces trying to split the cylinder along a length. The force due to the pressure is

$F = pA = pLD$ …(1.4)

So long as the material holds this is balanced by the stress in the material. The force due to the stress is

$F = \sigma_C$ multiplied by the area of the metal $= \sigma_C\, 2Lt$ …(1.5)

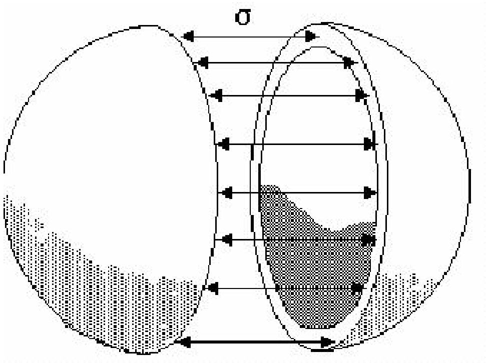
Equating 1.4 and 1.5 we have

$$\sigma_C = \frac{pD}{2t} \qquad \ldots(1.6)$$

It follows that for a given pressure the circumferential stress is twice the longitudinal stress.

THIN WALLED SPHERE

A sphere will tend to split about a diameter as shown in figure below.



The circumferential stress produced in the material is equivalent to the longitudinal stress in the cylinder so

$$\sigma_C = \frac{pD}{4t} \qquad \ldots(2.1)$$

EULER THEORY OF COLUMNS

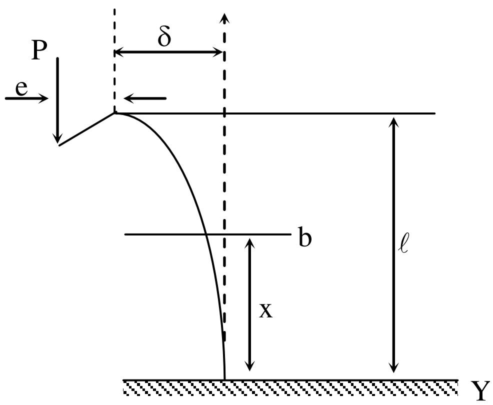
Compression members fall into three groups, long (struts), intermediate and short (columns). First we need to know that a material may fail due to exceeding the ultimate (maximum) compressive stress. This is often referred to as the crushing stress. If this was the only factor causing failure the load that produces it would be given by the formula $F_u = (\sigma_u \times A)$, where,

F_u = Ultimate compressive load
σ_u = ultimate compressive stress
A = cross-sectional area.

But bending due to compressions also has an effect on failure.

In discussing the bending of a slender column under the action of an eccentric load, the defection δ is important when there is eccentric loading with eccentricity e. Assuming that the eccentricity is in the plane of symmetry, the deflection occurs in the same axial plane xy in which the load P acts, and the bending at any cross-section ab is :

$$M = -P(\delta + e - y) \qquad ..(1)$$



The defection curve can be obtained from the above equation as.

$$EI_z \frac{d^2y}{dx^2} = P(\delta + e - y) \qquad \ldots(2)$$

Using,

$$\frac{P}{EI_z} = p^2 \qquad \ldots(3)$$

we get,

$$\frac{d^2y}{dx^2} + p^2 y = p^2(\delta + e) \qquad \ldots(4)$$

By substitution, we solve for y and obtain,

$$y = C_1 \sin px + C_2 \cos px + \delta + e. \qquad \ldots(5)$$

This solution contains two constants of integration C_1 and C_2, whose magnitudes must be adjusted so as to satisfy the conditions at the ends of the columns if we are to obtain the true deflection curve of the column. At the lower end, which is built in, the conditions are,

$$(y)\big|_{x=0} = 0, \left.\frac{dy}{dx}\right|_{x=0} = 0 \qquad \ldots(6)$$

Using these conditions and its first derivative we obtain

$$C_1 = 0,\ C_2 = -(\delta + e) \qquad \ldots(7)$$

Hence, the equation of the deflection curve becomes,

$$y = (\delta + e)(1 - \cos p x) \qquad \ldots(8)$$

The magnitude of the deflection δ at the upper end of the column is obtained by substituting $x = \ell$. The defection y on the left side must then be equal to δ and we obtain the equation,

$$\delta = (\delta + e)(1 - \cos p\ell) \qquad \ldots(9)$$

From which,

$$\delta = \frac{e(1 - \cos p\ell)}{\cos p\ell} \qquad \ldots(10)$$

From this we obtain the defection curve,

$$y = \frac{e(1 - \cos px)}{\cos p\ell} \qquad \ldots(11)$$

The maximum bending moment occurs at the built end of the column and has a magnitude

$$M_{max} = P(e + \delta) = Pe\sec(p\ell) \qquad \ldots(12)$$

CRITICAL LOAD : From the above equations we can see clearly that the deflection of an eccentrically compressed column increases very rapidly as the quantity $(p\ell)$ approaches $(\pi/2)$. When $(p\ell)$ becomes equal to $\pi/2$, the formulae for the deflections and for the maximum bending moment both give infinite values. To find the corresponding value of the load, we substitute $p = \pi/2\ell$ in the expression (3) and obtain,

$$P_{cr} = \frac{\pi^2 EI_z}{4\ell^2}$$

This value, which depends only on the dimensions of the column and the modulus of the material, is called the critical load or Euler's Load. The critical load is considered as an ultimate load which will produce complete failure of the column.

The expression of critical load above was desired for a column with one end built in and the other end free. Similar derivation can be made in the case of a strut with hinged ends. For a strut with hinged ends, the values of deflection δ and maximum bending moment is given by

$$\delta = \frac{e\left(1-\cos\left(\frac{p\ell}{2}\right)\right)}{Pe\,Sec\left(\frac{p\ell}{2}\right)}$$

$$M_{max} = PeSec\left(\frac{p\ell}{2}\right)$$

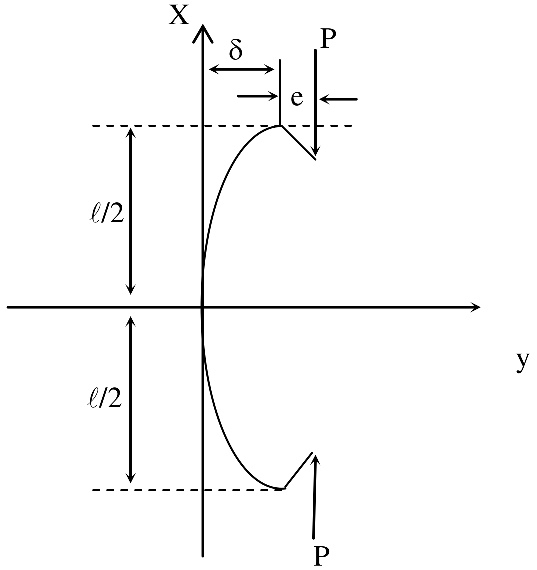


Fig Strut with Hinges Ends

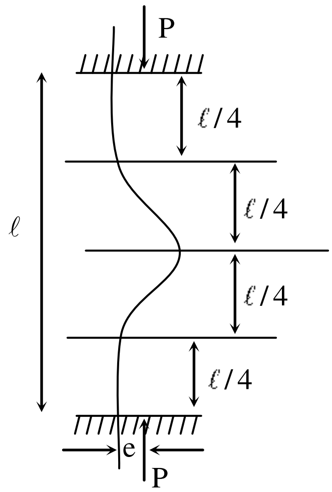
The equations of deflection and bending moment give infinite values when,

$$\frac{p\ell}{2} = \frac{\pi}{2}$$

Substituting this value for p in equation (3), we get

$$P_{cr} = \frac{\pi^2 EI_z}{\ell^2}$$

For the compression of columns with built ends as shown below



The critical value of the load is found by substituting $\ell/4$ instead of ℓ, giving

$$P_{cr} = \frac{4\pi^2 EI_z}{\ell^2}$$

This is the critical load for a column with built ends.

Similarly, for a column having one end fixed, and the other hinged is given try,

$$P_{cr} = \frac{2\pi^2 EI_z}{\ell^2}$$

Case (a) : Both ends hinged

$$P = \frac{\pi^2 EI}{L^2}$$

Case (b) : One end fixed, other end free

$$P = \frac{\pi^2 EI}{4\ell^2}$$

Case (c) : Both ends fixed

$$P = \frac{4\pi^2 EI}{\ell^2}$$

Case (d) : One end fixed, the other hinged

$$P = \frac{2\pi^2 EI}{\ell^2}$$

Limitations of Euler's Formula

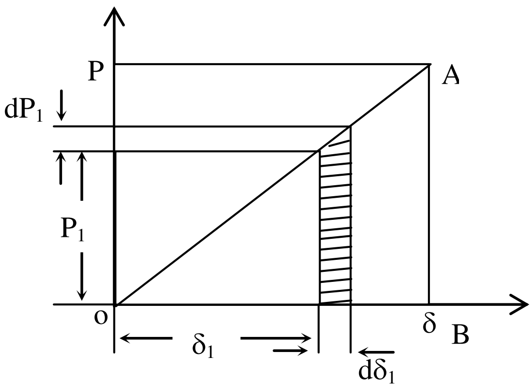
- The axis of the strut is perfectly straight when unloaded.
- The line of thrust coincides exactly with the unstrained axis of the strut.
- The flexural rigidity is uniform.
- The material is isotropic.
- The buckling value is assumed to obtain for all degrees of flexure.
- The slenderness ratio is greater than 80.4 for steel column hinged at both the ends.

STRAIN ENERGY METHODS

ELASTIC STRAIN ENERGY

For a bar in simple tension, during elongation under a gradually increasing load, work is done on the bar, and this work is transformed either partially or completely into potential energy of strain. If the strain remains within the elastic limit, the work done will be completely transformed into potential energy and can be recovered during a gradual unloading of the strained bar.

Let the final magnitude of the load P and the final elongation be δ. Let P_1 represent an intermediate value of the load and δ_1 be the corresponding elongation. An incremental dP_1 in the load causes an increment $d\delta_1$ in the elongation. The work done by P_1 during this elongation is $P_1\, d\delta_1$ as shown in the figure below.



The total work done in the process of loading when the load is increasing from O to P is given by the triangle OAB.
This represents the total energy U stored in the bar during loading. Then,

$$U = \frac{P\delta}{2} \qquad \ldots(1)$$

But we know that,

$$\delta = \frac{P\ell}{AE} \qquad \ldots(2)$$

$$\therefore \quad U = \frac{P^2\ell}{2AE} \qquad \ldots(3)$$

$$U = \frac{AE\delta^2}{2\ell} \qquad \ldots(4)$$

Equation (3) gives the strain energy as a function of the load P and the second gives the same energy as a function of the elongation δ.

The strain energy per unit volume is given by

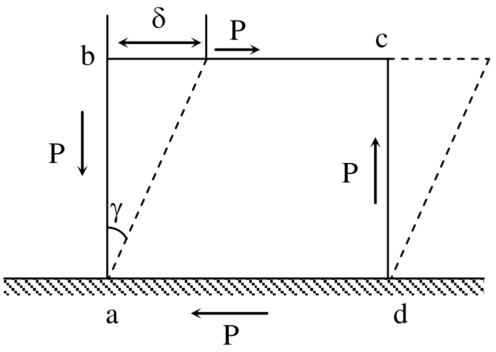
$$\omega = \frac{U}{A\ell} = \frac{\sigma^2}{2E} \quad \ldots(5)$$

$$\omega = \frac{E\in^2}{2} \quad \ldots(6)$$

Here $\sigma = P/A$ is the tensile stress and $e = \delta/\ell$ and $e = \delta/\ell$ is the unit elongation.

STRAIN ENERGY IN TENSION AND SHEAR

The strain energy stored in an element under pure shearing stress may be calculated by the method similar to that used for simple tension previously,



The work done by the force P and stored in the form of elastic strain energy is,

$$U = \frac{P\delta}{2} \quad \ldots(7)$$

Now,

$$\frac{\delta}{\ell} = \gamma = \frac{\tau}{G} = \frac{P}{AG} \quad \ldots(8)$$

$$U = \frac{P^2\ell}{2AG} \quad \ldots(9)$$

$$U = \frac{AG\delta^2}{2\ell} \quad \ldots(10)$$

The shearing strain energy per unit volume is given by

$$\omega = \frac{\tau^2}{2G} \quad \ldots(11)$$

$$\omega = \frac{\gamma^2 G}{2} \quad \ldots(12)$$

where $\tau = P/A$ is the shearing stress and $\gamma = \delta/\ell$ is the shearing strain

The energy stored in a twisted circular shaft is easily calculated by equation (11). If τ_{max} is the maximum shearing stress at the surface of the shaft, then τ_{max} (2r/d) is the shearing stress at a point a distance r from the axis, where d is the diameter of the shaft. The energy per unit volume at this point is,

$$\omega = \frac{2\tau_{max}^2 r^2}{Gd^2}$$

The energy stored in the material included between two cylindrical surfaces of radii r and r + dr is,

$$\frac{2\tau_{max}^2 r^2}{Gd^2}.2\pi\ell rdr.,$$

where ℓ is the length of the shaft. Then the total energy stored in the shaft is

$$U = \int_0^{d/2} \frac{2\tau_{max}^2 r^2}{Gd^2} 2\pi\ell rdr = \frac{1}{2}\frac{\pi d^2\ell}{4}\frac{\tau_{max}^2}{2G}$$

We see that the total energy is only half of what it would be if all elements of the shaft were stressed to the maximum shearing stress τ_{max} .

The energy of torsion can also be calculated based on the twisting moment M_t, and the angle of twist ϕ.

We know that,

$$\phi = \frac{M_t \ell}{GI_p}$$

$\therefore$ We obtain,

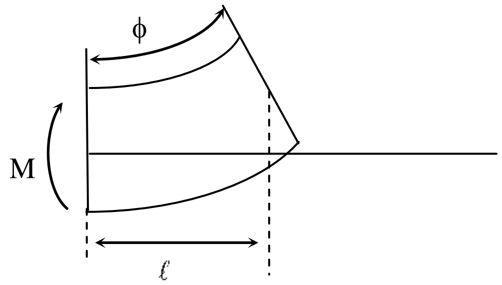
$$U = \frac{M^2{}_t \ell}{2GI_p}$$

$$U = \frac{\phi^2 GI_p}{2\ell}$$

ELASTIC STRAIN ENERGY IN BENDING

For a prismatic bar built in at one end, bent by a couple M applied at the other end and acting in one of the principal planes, the angular displacement at the free end is,

$$\phi = \frac{M\ell}{EI_z}$$



This displacement is proportional to the bending moment M, and the energy stored in the bar is,

$$U = \frac{M\phi}{2}$$

This energy may be expressed in the forms

$$U = \frac{M^2\ell}{2EI_z}$$

$$U = \phi^2 \frac{EI_z}{2\ell}$$

The potential energy can also be expressed as a function of the maximum normal stress $\sigma_{max} = M_{max}/Z$

Thus for a rectangular bar,

$$\sigma_{max} = \frac{6M}{bh^2}$$

$$M = \frac{bh^2\sigma_{max}}{6}, \text{ and}$$

$$U = \frac{1}{3} bh\ell \frac{\sigma^2{}_{max}}{2E}$$

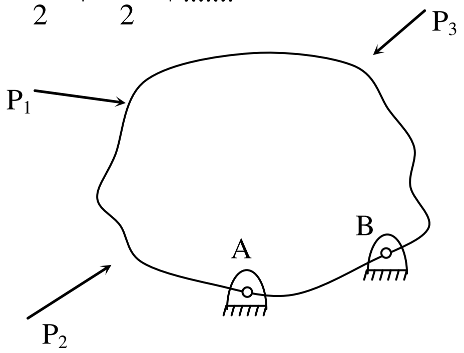
For a simply supported beam, at ends supported loads, the point of contraflexure occurs where the bending moment is zero.

GENERAL EXPRESSION FOR STRAIN ENERGY

Consider a body that is submitted to the action of external force P_1, P_2, P_3 …… and is supported in such a manner that movement as a rigid body is impossible and displacements are due to elastic deformations only. Let δ_1, δ_2, δ_3…… denote the displacements of the points of application of the forces, each measured in the direction of the force.

The work done by the forces is equal to the strain energy stored in the deformed body which is given as,

$$U = \frac{P_1\delta_1}{2} + \frac{P_2\delta_2}{2} + \frac{P_3\delta_3}{2} + ...$$



CASTIGLIANO THEOREM

This method is used to calculate the displacements of points of an elastic body during deformation using Strain Energy. The strain energy is given by.

$$U = \frac{P^2\ell}{2AE}$$

By taking the derivative of this expression with respect to P, we get

$$\frac{dU}{dP} = \frac{P\ell}{AE} = \delta$$

Hence the derivative of the strain energy w.r.t. the load gives the displacement corresponding to the load in the direction of the load.

Volumetric Strain Energy

Denoted by U, it is the work done by the stresses in straining the material. It is sufficiently general to consider a unit cube acted on by the principal stresses. The total work done is then given as:

$$U_T = \frac{1}{2}\sigma_x\varepsilon_x + \frac{1}{2}\sigma_y\varepsilon_y + \frac{1}{2}\sigma_z\varepsilon_z + \frac{1}{2G}\left(\tau_{xy}{}^2 + \tau_{yz}{}^2 + \tau_{zx}{}^2\right)$$

$$U_T = \frac{1}{2E}\left[\sigma_x{}^2 + \sigma_y{}^2 + \sigma_z{}^2 - 2v\left(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x\right)\right] + \frac{1}{2G}\left(\tau_{xy}{}^2 + \tau_{yz}{}^2 + \tau_{zx}{}^2\right)$$

Note that:

$$\frac{1}{2}\sigma_x\varepsilon_x = \frac{1}{2}\sigma_x\left(\frac{\sigma_x}{E} - \frac{v\sigma_y}{E} - \frac{v\sigma_z}{E}\right)$$

$$\frac{1}{2}\sigma_y\varepsilon_y = \frac{1}{2}\sigma_y\left(\frac{\sigma_y}{E} - \frac{v\sigma_x}{E} - \frac{v\sigma_z}{E}\right)$$

$$\frac{1}{2}\sigma_z\varepsilon_z = \frac{1}{2}\sigma_z\left(\frac{\sigma_z}{E} - \frac{v\sigma_x}{E} - \frac{v\sigma_y}{E}\right)$$

$$\therefore \quad U_T = \frac{1}{2E}\left[\sigma_1{}^2 + \sigma_2{}^2 + \sigma_3{}^2 - 2v\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)\right]$$

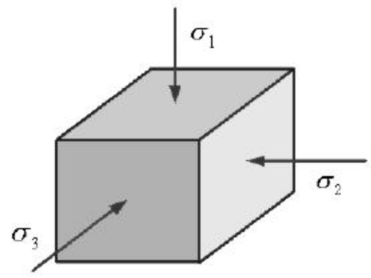
Volumetric Strain Energy

It is the energy required to deform a body in 3D and is derived as follows—
The principal stresses are:

$$\sigma_1 = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) + \frac{1}{3}(\sigma_1 - \sigma_2) + \frac{1}{3}(\sigma_1 - \sigma_3)$$

$$\sigma_2 = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) + \frac{1}{3}(\sigma_2 - \sigma_1) + \frac{1}{3}(\sigma_2 - \sigma_3)$$

$$\sigma_3 = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) + \frac{1}{3}(\sigma_3 - \sigma_1) + \frac{1}{3}(\sigma_3 - \sigma_2)$$



The volumetric strain energy per unit volume in 3-D

$$U_V = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2v(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] + 0$$

$$U_v = \frac{3}{2E}\left[\left(\frac{\sigma_1 + \sigma_2 + \sigma_3}{3}\right)^2 (1 - 2v)\right]$$

Volumetric Shear Strain Energy

It is the algebraic sum of volumetric strain energy per unit volume and shear strain energy per unit volume in terms of principal stress.

$$U_S = \left(\frac{1+v}{6E}\right)\left[2(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 2(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

$$U_s = \frac{1}{12G}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]$$

Theories of failure

a) Maximum Principal Stress Theory / Rankine Theory :
Applied satisfactorily to many brittle materials, the theory is based on a limiting normal stress. Failure occurs when the normal stress reaches a specified upper limit.

According to this theory failure will occur when the maximum principal stress in the complex system reaches the value of the maximum stress at the elastic limit in simple tension.

$$\sigma_1 = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}\sqrt{\left[(\sigma_x - \sigma_y)^2 + 4\tau^2\right]}$$

$$= \sigma \qquad \text{(in simple tension)}$$

b) Maximum Shear Stress Theory :
Independent of the complexity of the stress state, yielding is assumed to occur when the maximum shearing stress in the material reaches a value equal to the maximum shearing stress for the material. For a plane stress state where the two in-plane principal stresses are of opposite sign the maximum shear stress is given by:

$$\frac{1}{2}(\sigma_2 - \sigma_1) = \frac{1}{2}\sqrt{\left[(\sigma_x - \sigma_y)^2 + 4\tau^2\right]}$$

$$= \frac{1}{2}\sigma \qquad \text{(in simple tension)}$$

If the in-plane principal stresses are of the same sign then we must consider the third principal stress. The third principal stress may be the maximum, minimum or intermediate principal stress.

In a thin-walled pressure vessel for example, the in-plane principal stresses are both positive and the minimum normal stress acts normal to the surface of the pressure vessel.

This theory is appealing since for some ductile materials (e.g. hot-rolled carbon steel) we can observe slip occurring at orientations which appear to agree with the maximum shear planes. Recall the orientation of the slip planes from your tensile and torsion tests of hot rolled carbon steel. This theory is quite simple to apply and gives reasonable results when applied to many ductile materials subjected to fairly simple loading states.

c) Strain Energy Theory:
Applicable to many types of materials, the theory predicts failure or inelastic action at a point when the strain energy per unit volume exceeds a specified limit.
Failure is predicted when the total strain energy associated with the principal stresses, $_{1,2}$, equals or exceeds the total strain energy corresponding to that for the yield strength, $_{yp}$, of the material in uniaxial tension or compression.

$$\left(\frac{1}{2E}\right)\left[\sigma_1^2+\sigma_2^2+\sigma_3^2-2\nu\left(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1\right)\right]=\frac{\sigma^2}{2E}$$

$$\Rightarrow\quad \sigma^2=\sigma_1^2+\sigma_2^2+\sigma_3^2-2\nu\left(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1\right)$$

d) Shear Strain Energy Theory (Mises and Hencky Theory):
The theory is based on a limiting energy of distortion, i.e. energy associated with shear strains.
Strain energy can be separated into energy associated with volume change and energy associated with distortion of the body. The maximum distortion energy failure theory assumes failure by yielding in a more complicated loading situation to occur when the distortion energy in the material reaches the same value as in a tension test at yield. This theory provides the best agreement between experiment and theory and, along the Tresca theory, is very widely used today.

$$\frac{1}{12G}\left[\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2\right]=\frac{\sigma^2}{6G}$$

$$\Rightarrow\quad 2\sigma^2=\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2$$

e) Maximum Principal Strain Theory (Saint Venant Theory):
The theory is based on the assumption that inelastic behavior or failure is governed by a specified maximum normal strain. Failure will occur at a particular part in a body subjected to an arbitrary state of strain when the normal strain reaches a limiting level.
Failure is predicted when either of the principal strains, resulting from the principal stresses, $_{1,2}$, equals or exceeds the maximum strain corresponding to the yield strength, $_{yp}$, of the material in uni-axial tension or compression

$$\varepsilon_1=\left(\frac{1}{E}\right)\left(\sigma_1-\nu\sigma_2-\nu\sigma_3\right)=\frac{\sigma}{E}\qquad\text{(in simple tension)}$$

$$\Rightarrow\quad \sigma=\sigma_1-\nu\sigma_2-\nu\sigma_3$$

THERMAL STRESS

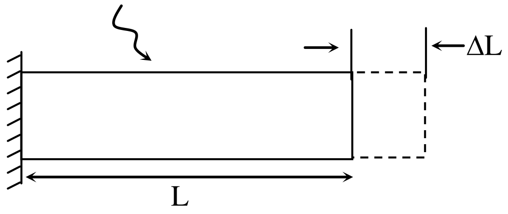
Thermal stress is the stress induced in a body as a result of the body being unable to thermally expand or contract due to a temperature change by virtue of being constrained. The change in the temperature does not lead to the normal free expansion or contraction and there is a mechanical strain induced in the body.

Consider a bar with built ends. If the temperature of the bar is raised from T_0 to T and thermal expansion is prevented by the reactions at the ends, compressive stresses are produced in the bar whose magnitude may be calculated from the condition that the length remains unchanged. If α denotes the coefficient for thermal expansion and σ denotes the compressive stresses produced by the reactions, then the equation for determining σ will be,

$$\alpha\,(T - T_o) = \frac{\sigma}{E}$$

from which

$$\sigma = E\,\alpha\,(T - T_o)$$



For a long rod the main thermal deformation occurs along the length of the rod, and is given by:

$$\delta = \alpha\,(\Delta T)\,L$$

where α(alpha) is the linear coefficient of expansion for the material, and is the fractional change in length per degree change in temperature. **[Some values of the linear coefficient of expansion are: Steel = 12 × 10^{-6}/°C = 6.5 × 10^{-6}/°F; Brass = 20 × 10^{-6}/°C = 11 × 10^{-6}/°F; Aluminum = 23 × 10^{-6}/°C = 13 × 10^{-6}/°F.]** The term ΔT is the temperature change the material experiences, which represents (T_f - T_o), the final temperature minus the original temperature. If the change in temperature is positive we have thermal expansion, and if negative, thermal contraction. The term 'L' represents the initial length of the rod.

LIST OF FORMULAE

- Stress

$$\sigma = \frac{P}{A}$$

$$(\sigma_n)_{max} = \sigma_x$$

$$(\tau)_{max} = \frac{1}{2}\sigma_x$$

$$\sigma_\theta = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_x - \sigma_y)\cos 2\theta}{2} + \tau_{xy}\sin 2\theta$$

$$\tau_\theta = \frac{(\sigma_x - \sigma_y)\sin 2\theta}{2} - \tau_{xy}\cos 2\theta$$

- Principal Stress

$$\left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{max} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\sigma_{min} = \frac{\sigma_1 + \sigma_2}{2}$$

$$\tan 2\theta = \left(\frac{2\tau}{\sigma_y - \sigma_x}\right)$$

- Strain

$$\text{strain} = \frac{\text{extension}}{\text{length}} = \frac{\Delta L}{L}$$

$$\text{Volumetric strain} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = (\sigma_1 + \sigma_2 + \sigma_3)\left[\frac{1 - 2\nu}{E}\right]$$

- Relation between stress and strain

  i) $\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right)$ ii) $\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\sigma_x\right)$

  iii) $\sigma_1 = \left(\frac{E}{1-v^2}\right)\left(\varepsilon_1 + v\varepsilon_2\right)$ iv) $\sigma_2 = \left(\frac{E}{1-v^2}\right)\left(\varepsilon_2 + v\varepsilon_1\right)$

- Relationship between the elastic constants

$$\varepsilon_{vol} = \frac{P}{E}\left(\frac{1-2}{1/\nu}\right)$$

Young's modulus :

$$E = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta L/L}$$

$$\frac{\varepsilon_x}{\varepsilon_y} = -\nu$$

$$K = \frac{E}{3(1-2\nu)}, \qquad G = \frac{E}{2(1+\nu)}$$

- Mohr's Circle

$$\sigma_{max} = \sigma_1 = \frac{\left(\sigma_x + \sigma_y\right)}{2} + \frac{\sqrt{\left(\sigma_x - \sigma_y\right)^2 + 4\tau_{xy}^2}}{2}$$

$$\sigma_{min} = \sigma_2 = \frac{\left(\sigma_x + \sigma_y\right)}{2} - \frac{\sqrt{\left(\sigma_x - \sigma_y\right)^2 + 4\tau_{xy}^2}}{2}$$

$$\tau_{max} = \frac{\sqrt{\left(\sigma_x - \sigma_y\right)^2 + 4\tau_{xy}^2}}{2} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{min} = \frac{-\sqrt{\left(\sigma_x - \sigma_y\right)^2 + 4\tau_{xy}^2}}{2} = -\left[\frac{\sigma_1 - \sigma_2}{2}\right]$$

$$\text{Centre} = \left(\frac{\sigma_x + \sigma_y}{2}, 0\right)$$

$$\text{The radius of circle} = r = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$

- Relation between Bending Moment and Shearing Force :

$$\frac{dM}{dx} = V$$

$$\frac{dV}{dx} = -q$$

$$\frac{d^2M}{dx^2} = -q$$

- Flexure Formula

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

- Radius of curvature in Bending

$$\frac{1}{R} = -\frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$

- Torsion of circular shaft

$$\frac{T}{I_p} = \frac{\tau}{r} = \frac{G\theta}{L}$$

$$\gamma = \frac{1}{2}\theta d$$

$$\tau = \frac{1}{2}G\,\theta d$$

$$\phi = \theta\ell = \frac{M_t\ell}{GI_p}$$

$$\tau_{max} = \frac{M_t d}{2I_{p.}}$$

$$T = \frac{1000}{2\pi f}P = \frac{159\times P}{f}$$ where P is in KW.

- Thin Walled Cylinder

$$\sigma_L = \frac{pD}{4t}$$

$$\sigma_C = \frac{pD}{2t}$$

- Thin Walled Sphere

$$\sigma_C = \frac{pD}{4t}$$

- Euler Theory of Columns

$$P_{cr} = \frac{2\pi^2 EI_z}{\ell^2}$$

Case (a) : Both ends hinged

$$P = \frac{\pi^2 EI}{L^2}$$

Case (b) : One end fixed, other end free

$$P = \frac{\pi^2 EI}{4\ell^2}$$

Case (c) : Both ends fixed

$$P = \frac{4\pi^2 EI}{\ell^2}$$

Case (d) : One end fixed, the other hinged

$$P = \frac{2\pi^2 EI}{\ell^2}$$

- Elastic Strain Energy

$$U = \frac{P^2 \ell}{2AE}$$

- Strain Energy in Tension and Shear

$$U = \frac{AG\delta^2}{2\ell}$$

$$U = \frac{\phi^2 GI_p}{2\ell}$$

- Elastic Strain Energy in Bending

$$U = \phi^2 \frac{EI_z}{2\ell}$$

- Castigliano Theorem

$$U = \frac{P^2 \ell}{2AE}$$

$$\frac{dU}{dP} = \frac{P\ell}{AE} = \delta$$

- Volumetric Strain Energy

$$U_T = \frac{1}{2E}\left[\sigma_x{}^2 + \sigma_y{}^2 + \sigma_z{}^2 - 2v\left(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x\right)\right] + \frac{1}{2G}\left(\tau_{xy}{}^2 + \tau_{yz}{}^2 + \tau_{zx}{}^2\right)$$

$$U_v = \frac{3}{2E}\left[\left(\frac{\sigma_1 + \sigma_2 + \sigma_3}{3}\right)^2 (1 - 2v)\right]$$

- Volumetric Shear Strain Energy

$$U_s = \frac{1}{12G}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]$$

- Theories of failure

  a) Maximum Principal Stress Theory / Rankine Theory :

$$\sigma_1 = \frac{1}{2}\left(\sigma_x + \sigma_y\right) + \frac{1}{2}\sqrt{\left[\left(\sigma_x - \sigma_y\right)^2 + 4\tau^2\right]}$$

  b) Maximum Shear Stress Theory :

$$\frac{1}{2}(\sigma_2 - \sigma_1) = \frac{1}{2}\sqrt{\left[\left(\sigma_x - \sigma_y\right)^2 + 4\tau^2\right]}$$

  c) Strain Energy Theory:

$$\sigma^2 = \sigma_1{}^2 + \sigma_2{}^2 + \sigma_3{}^2 - 2\nu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)$$

  d) Shear Strain Energy Theory (Mises and Hencky Theory):

$$2\sigma^2 = \left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2$$

  e) Maximum Principal Strain Theory (Saint Venant Theory):

$$\sigma = \sigma_1 - \nu\sigma_2 - \nu\sigma_3$$

- Thermal stress

$$\sigma = E\,\alpha\,(T - T_o)$$

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ASSIGNMENT – 1

Duration : 45 Min. **Max. Marks : 30**

Q1 to Q6 carry one mark each

1. In case of non-dilatent materials, the maximum value of Poisson's ratio is:
(A) 0.25 (B) 0.33
(C) 0.47 (D) 0.5

2. If S is the elongation produced in a bar of length L, diameter D and cross−sectional area A, by a direct load of W, then Young's modulus may be given by the following:
(A) E = WL/SA (B) S = WE/A
(C) W = SDE/A (D) A = WSL/E

3. The numerical value of Young's modulus of elasticity in ascending order of glass, aluminium, copper, wrought iron and tungsten are given in:
(A) Tungsten, wrought iron, copper, aluminium, glass
(B) wrought iron, copper, aluminium, glass, tungsten
(C) copper, aluminium, glass, tungsten, wrought iron
(D) glass, aluminium, copper, wrought iron, tungsten

4. Strain is:
(A) The change in metals
(B) The change in shape and direction in metals
(C) The result of load application on a body
(D) The change of form produced in a piece by the action of load

5. Tensile stress is:
(A) Stress due to any force
(B) Stress due to change in length under a load
(C) Stress caused by varying load
(D) Stress measured by the ratio of the increase or decrease in length of the unloaded piece under tensile force.

6. A material capable of absorbing large amounts of energy is known as:
(A) Ductile (B) Shock proof
(C) Resilient (D) Hard

Q7 to Q18 carry two marks each

7. The ratio of bulk modulus to shear modulus for Poisson's ratio 0.25 is:
(A) 3/2 (B) 5/3
(C) 1 (D) 3/5

8. The stress–strain curve for a glass rod during tensile test would exhibit:
(A) A straight line (B) A parabola
(C) A sudden break (D) An irregular curve

9. A cylindrical bar of L meters length deforms by ℓ mm. The strain in the bar will be:
(A) ℓ / L (B) 0.1 ℓ / L
(C) 0.01 ℓ / L (D) 0.001 ℓ / L

10. Materials exhibiting time bound behaviour are known as:
(A) isentropic (B) reactive
(C) fissile (D) visco elastic

11. If Young's modulus of elasticity is determined for mild steel in tension and compression, the two values will have a ratio (E_t/E_c) of:
(A) 1 (B) 1.2
(C) 1.4 (D) 0.8

12. Poisson's ratio for cast iron is:
(A) 0.27 (B) 0.31
(C) 0.33 (D) 0.36

13. A body having similar properties throughout its volume is said to be:
(A) Homogenous (B) Isotropic
(C) Isentropic (D) Continuous

14. A beam of uniform strength is one in which:
(A) The cross-section remains same throughout
(B) The bending moment is same at every section
(C) The stiffness is same at every section
(D) The bending stress is same at every section

15. It is desired to punch a hole of 20mm diameter in a plate 20mm thick. If the shear stress of mild steel is 30kg/mm^2 the force necessary for punching would be approximately in the range:
(A) 10-15 tonnes (B) 15-20 tonnes
(C) 20-25 tonnes (D) 35-45 tonnes

16. A solid uniform metal bar of diameter D and length L is hanging vertically from its upper end. If W is the total weight of the bar and E is the Young's modulus of elasticity, the elongation of the bar due to self weight would be:
(A) WL/2AE (B) 2WL/AE
(C) WL/AE (D) 4WL/AE

17. The relation between modulus of elasticity E, modulus of elasticity in shear G and Poisson's ratio v is:
(A) $E = Gv$ (B) $E = G(v + 1)$
(C) $E = 2G(v + 1)$ (D) $E = 4G(1 + 2v)$

18. The relationship between modulus of elasticity E and Bulk modulus of elasticity K is:
(A) $K = E/(1 - 2v)$ (B) $K = E/(3 - 2v)$
(C) $K = E/(3(1 - v)$ (D) $K = E/3(1 - 2v)$

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ASSIGNMENT – 2

Duration : 45 Min. **Max. Marks : 30**

Q1 to Q6 carry one mark each

1. Unsymmetrical bending is said to occur when:
(A) The beam cross-section is unsymmetrical
(B) The shear center does not coincide with neutral axis
(C) The bending moment diagram is unsymmetrical
(D) The beam is subjected to loads that do not lie in a plane containing a principal axis

2. If shear force along a section is zero the bending moment at that section will be :
(A) zero (B) minimum
(C) maximum (D) maximum or minimum

3. A pin type of support is capable of resisting
(A) force in direction coincident with line of action
(B) force in direction ⊥ to line of action
(C) Moment
(D) Force in any direction

4. The plane of maximum shear stress has normal stress that is
(A) maximum (B) minimum
(C) maximum or minimum (D) zero

5. Mohr's circle reduces to a point when the body is subjected to:
(A) Pure shear
(B) Uniaxial stress only
(C) Equal and opposite axial stresses on two mutually perpendicular planes, the planes being free of shear
(D) Equal axial stresses on two mutually perpendicular planes, the planes being free of shear

6. In the Mohr's circle, the radius is equal to the:
(A) Maximum shear stress
(B) Maximum principal stress
(C) Minimum principal stress
(D) Zero

Q7 to Q18 carry two marks each

7. A cantilever beam of diameter D, length L, cross–section A is subjected to a uniformly distributed load W and a concentrated load W_1 at a distance L_1 from the free end will have maximum bending moment of:
(A) $WL/2 + W_1L_1$ (B) $WL^2/2 + W_1L_1$
(C) $W(L - L_1) + W_1(L - L_1)$ (D) $WL/2 + W_1(L - L_1)$

8. A simply supported beam of length L carrying a load W concentrated at the center of span will have maximum bending moment of:
(A) WL (B) WL/2
(C) WL/4 (D) WL/8

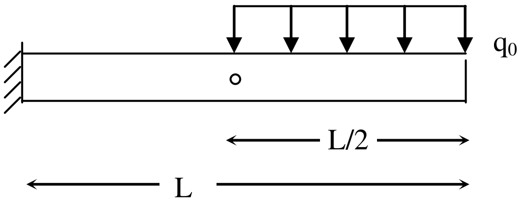
9. For a beam of uniform strength, if its depth is maintained constant, then its width will vary in proportion to:

(A) Bending moment, BM (B) BM^2
(C) $BM^{1/3}$ (D) BM^3

10. If a simple supported of diameter D, cross–section A, length L is subjected to uniformly distributed load W, it will have maximum bending moment equal to :

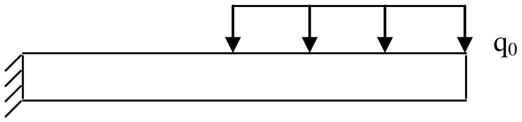
(A) $\frac{WL^2}{8}$ (B) $\frac{WLA}{8}$
(C) $\frac{WL^2}{2A}$ (D) $\frac{WL}{8}$

11. For a cantilever as loaded below, the maximum shear force at any point is :

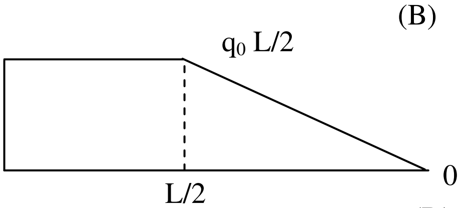


(A) $\frac{q_0L}{2}$ (B) q_0L
(C) $\frac{q_0L^2}{2}$ (D) $\frac{q_0L}{52}$

12. For a cantilever as loaded below the shear force diagram would look like :

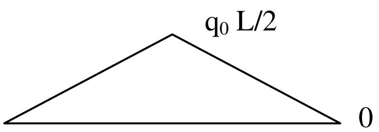


(A)

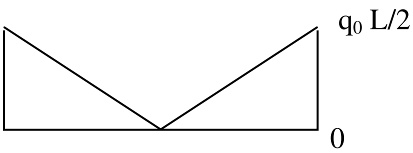


(B)

(C)



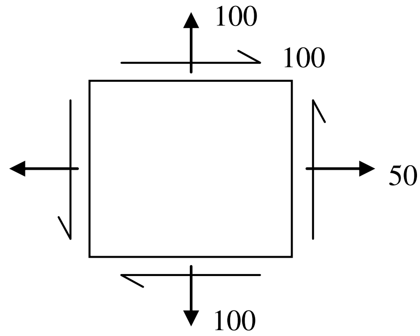
(D)



13. Shear stress distribution diagram of a beam of rectangular cross–section subjected to transverse loading will be

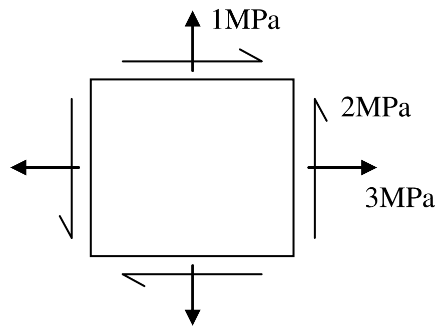
(A) Parabola (B) Rectangular
(C) Triangle (D) Square

14. For the given block and stresses, the principal stress will be (values given in MPa) :



(A) 96.45MPa (B) 156.72MPa
(C) 43.7MPa (D) 103.07MPa

15. For the given block and stresses, the tensile stress at an angle −22.5 degrees from the horizontal are:



(A) 3.45MPa
(B) 2.12MPa
(C) 1.29MPa
(D) 7.34MPa

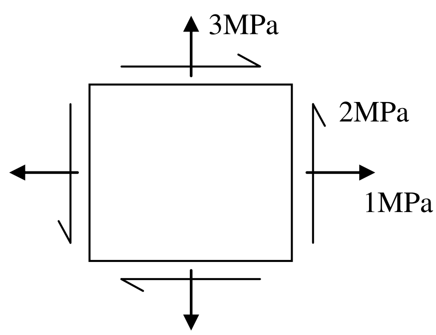
16. In the above problem, the shear stress would be:
(A) 3.45MPa (B) 2.12MPa
(C) 1.29MPa (D) 7.34MPa

17. A state of pure shear in a biaxial state of stress is given by :

(A) $\begin{bmatrix} \sigma_1 & 0 \\ \sigma_2 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} -\sigma_1 & 0 \\ 0 & -\sigma_2 \end{bmatrix}$

(C) $\begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$ (D) $\begin{bmatrix} \sigma_1 & \tau_{xy} \\ \tau_{xy} & \sigma_2 \end{bmatrix}$

18. For the given loading of stress, the maximum shear stress would be:



(A) 4.48MPa (B) 2.24MPa
(C) 1.12MPa (D) 3.36MPa

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ASSIGNMENT – 3

Duration : 45 Min. **Max. Marks : 30**

Q1 to Q6 carry one mark each

1. For a hollow sphere, the ratio longitudinal stress/ hoop stress is:

(A) 1/2 (B) 1
(C) 2 (D) 4

2. In the case of thin cylindrical shells with hemispherical ends:

(A) the thickness of cylindrical is more than that of spherical ends
(B) the thickness of cylindrical shell is less than that of spherical ends
(C) the thickness of cylindrical shell is same as that of spherical ends
(D) the thickness of cylindrical shell is double that of spherical ends

3. In design of a shaft, pulley and key for a system:
(A) Shaft is the weakest member
(B) Pulley is the weakest member
(C) Key is the weakest member
(D) All are designed equally strong

4. The value of J in the equation $T/J = \tau/r = G\theta/L$ for a circular solid shaft of diameter D will be:
(A) $\pi D^4/32$ (B) $\pi D^2/32$
(C) $\pi D^8/32$ (D) $\pi D^4/16$

5. In case of a circular shaft subjected to torque, the value of shear stress:
(A) Is uniform throughout
(B) Has maximum value at the axis
(C) Has maximum value at the surface
(D) Is zero at the axis and linearly increases to a maximum value at the surface of the shaft

6. Torsional rigidity is:
(A) The torque that produces rotation of unit r.p.m in a shaft
(B) The torque that transmits one HP at one rpm
(C) The torque that produces a twist of one radian in a shaft of unit diameter
(D) The torque that produces a twist of one radian in a shaft of unit length

Q7 to Q18 carry two marks each

7. Consider a closed cylindrical steel pressure vessel. The radius of the cylinder is 1000 mm and its wall thickness is 10 mm. The hoop and longitudinal stresses in the cylindrical wall for an internal pressure of 0.8 MPa would be:
(A) 60 MPa, 80 MPa (B) 80 MPa, 60 MPa
(C) 80 MPa, 40 MPa (D) 40 MPa, 80 MPa

8. In the above question, the change in diameter of the cylinder caused by pressurization would be:
(A) 0.4 mm (B) 0.3 mm
(C) 0.2 mm (D) 0.35 mm

9. Consider a steel spherical vessel of radius 1000 mm having a wall thickness of 10 mm. The maximum membrane stresses caused by an internal pressure of 0.8 MPa would be:
(A) 60 MPa (B) 40 MPa
(C) 80 MPa (D) 100 MPa

10. In the above question, the change in diameter in the sphere caused by pressurization would be:
(A) 0.3 mm (B) 0.45 mm
(C) 0.5 mm (D) 0.15 mm

11. An axial load is applied on a circular section of diameter D. If the same load is applied on a hollow circular section with inner diameter D/2, the ratio of stress in the two cases would be:
(A) 4/3 (B) 1
(C) 3/4 (D) 1/2

12. An axial load P is applied on a circular section of diameter D. If the same load is applied on a hollow circular shaft with inner diameter D/3, the ratio of the stresses in the two cases would be:
(A) 4/3 (B) 9/8
(C) 1 (D) 8/9

13. Find the maximum torsional shear stress in a shaft having an internal maximum torque resisted by the shaft equaling 40 N-m. The shaft is 8mm in diameter.
(A) 500MPa (B) 450MPa
(C) 400MPa (D) 350MPa

14. A long tube of 20 mm outside diameter, and 16 mm inside diameter is twisted about its longitudinal axis with a torque of 40N-m. The shear stress at the outside of the tube is:
(A) 47.8 MPa (B) 23.8 MPa
(C) 76.2 MPa (D) 43.1 MPa

15. In the above question, the shear stress at the inside of the tube would be:
(A) 56.1 MPa (B) 40.3 MPa
(C) 34.5 MPa (D) 73 MPa

16. A solid shaft has maximum shear stress of 70MPa. The diameter of the shaft for a 150KW motor operating at 0.3 Hz is around:
(A) 180 mm (B) 150 mm
(C) 130 mm (D) 170 mm

17. A solid shaft has maximum shear stress of 70MPa. The diameter of the shaft for a 150KW motor operating at 300 Hz is around:
(A) 10 mm (B) 15 mm
(C) 18 mm (D) 13 mm

18. The ratio of bending to twisting moments to produce bending and shear stresses of equal magnitudes is:
(A) 1/4 (B) 1/2
(C) 1 (D) 2

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ASSIGNMENT – 4

Duration : 45 Min. **Max. Marks : 30**

Q1 to Q6 carry one mark each

1. Euler's formula for a column of length fixed at one end and free at the other is:
(A) $\pi^2/4\ EL/n\ell^2$ (B) $\pi^2/2\ EL/n\ell^2$
(C) $\pi^2\ EL/n\ell^2$ (D) $2\pi^2\ EL/n\ell^2$

2. Euler's formula for a column of length ℓ, with both ends free but guided in the direction of load is:
(A) $\pi^2/4\ EL/n\ell^2$ (B) $\pi^2/2\ EL/n\ell^2$
(C) $\pi^2\ EL/n\ell^2$ (D) $2\pi^2\ EL/n\ell^2$

3. For which case is the buckling load a maximum:
(A) Both ends of the column are hinged
(B) One end is fixed and the other end is free
(C) One end is fixed and the other end is hinged
(D) Both ends are fixed

4. The strain for a body having coefficient of linear expansion α and change in temperature ΔT is:
(A) $\dfrac{\alpha}{\Delta T}$ (B) $2\alpha\ \Delta T$
(C) $\alpha\ \Delta T$ (D) $\Delta T / 2\alpha$

5. Failure occurring when the maximum principal stress in the complex system reaches the value of the maximum stress at the elastic limit in simple tension is known as:
(A) Rankine theory (B) Maximum shear stress theory
(C) Strain energy theory (D) Von–Mises theory

6. Strain energy per unit volume is also known as:
(A) Resilience (B) Volumetric strain energy
(C) Shear strain energy (D) Torsional energy

Q7 to Q18 carry two marks each

7. A rod of steel (E = 200 GPa and $\alpha = 12 \times 10^{-6}\ °C^{-1}$) is fixed at both ends. There is no stress when the temp is 35°C. The axial stress when the temperature is increased to 85°C is :
(A) 50 MPa (T) (B) 50 MPa (C)
(C) 120 MPa (C) (D) 120 MPa (T)

8. A copper bar of 25 cm length is fixed by means of supports at its ends. Supports can yield by 0.01 cm. If the temp of the bar is raised by 100°C, then the stress induced in the bar for
$\alpha = 2 \times 10^{-6}$ /°C and $E = 1 \times 10^{6}$ kg /cm^2 is
(A) 0 kg/cm^2 (B) 400 kg/cm^2
(C) 20 kg/cm^2 (D) 80 kg/cm^2

9. A steel rod of 200 cm length is fixed at both ends. If the temperature of the bar is raised by 30°C, then the stress induced in the bar for $\alpha = 12 \times 10^{-6}$ /°C and E = 200 GPa is
(A) 48 MPa (T) (B) 72 MPa (C)
(C) 84 MPa (C) (D) 120 MPa (T)

10. For a given copper rod, the $\alpha = 2\times10^{-6}/°C$ and E = 1 × 10^6 kg/cm^2.
For a compressive stress = 100 kg/cm^2 the change in temperature would be :
(A) 50°C (inc) (B) 85°C (dec)
(C) 70°C (inc) (D) 25°C (dec)

11. Two rods, one of steel (α = 12 × 10^{-6} /°C) and other of copper (α = 2 × 10^{-6} /°C) are of the same length and free at one end. When treated through the same ΔT,
(A) there is equal tension in both rods
(B) there is equal compression in both rods
(C) tension in steel rod is more than that in copper
(D) compression in steel rod is more than that in copper

12. The strain energy (J) of an axially loaded bar of length 1m, loaded with a force 10N and having A = 0.1 m^2, E = 2 × 10^9 Pa is
(A) 5 × 10^{-9} (B) 10^{-8}
(C) 10^{-9} (D) 2.5 × 10^{-9}

13. An axially loaded bar having strain energy = 10 × 10^{-9} has length 1m, A = 0.01 m^2 and E = 20 GPa will have an elongation of :
(A) 10^{-6} mm (B) 10^{-8} mm
(C) 10^{-8} m (D) 10^{-6} m

14. For a beam undergoing bending the strain energy dU is :
(A) $2EIM^2$ (B) $\frac{M^2}{4EI}$
(C) $4EIM^2$ (D) $\frac{M^2}{2EI}$

15. A cantilever is acted upon by a force P at the free end. Length of the cantilever is L, area of cross–section is A, modulus of elasticity is E. The strain energy stored would be :
(A) $\frac{P^2L^3}{3EI}$ (B) $\frac{P^2L^3}{2EI}$
(C) $\frac{P^2L^3}{6EI}$ (D) $\frac{P^2L^2}{3EI}$

16. The formula for elastic strain energy for a circular tube in torsion is :
(A) $dU = \frac{G^2}{2TI}$ (B) $dU = \frac{T^2}{2TI}$
(C) $dU = \frac{T^2}{2GI}$ (D) $dU = \frac{G^2}{4TI}$

17. In an elastic material, the principal stresses at a point are 60N/mm^2, 50 N/mm^2 and 20N/mm^2. The strain energy for v = 0.2 and E = 100000N/mm^2 is:
(A) 425 x 10^{-4} (B) 369 x 10^{-5}
(C) 221 x 10^{-4} (D) 417 x 10^{-4}

18. For a given plane in the complex system, the stresses are 40N/mm^2 and 20N/mm^2, and the shear is 60 N/mm^2. The maximum stress at the elastic limit in simple tension as per Rankine's theory is:
(A) 36 N/mm^2 (B) 24N/mm^2
(C) 91N/mm^2 (D) 172N/mm^2

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ASSIGNMENT – 5

Duration : 45 Min. **Max. Marks : 30**

Q1 to Q6 carry one mark each

1. The neutral axis of a beam:
(A) Is subjected to maximum stress
(B) Is subjected to maximum shear force
(C) Has tensile stress on one side and compressive stress on the other
(D) Is in the same plane in which the beam bends

2. A horizontal beam has σ_{max} = 300 MPa at a given section, E = 200 GPa and height of beam is 2mm thick. The radius of curvature is :
(A) 2000 mm (B) 1200 mm
(C) 666.67 mm (D) 1000 mm

3. The correct relation between the curvature and the deflection v of the beam is :
(A) $\frac{1}{R} = \frac{v'''}{\left[1+(v'')^2\right]^2}$ (B) $\frac{1}{R} = \frac{v''}{\left[1+(v')^2\right]^{3/2}}$
(C) $\frac{1}{R} = \frac{v''}{\left[1+(v'')^2\right]^{3/2}}$ (D) $\frac{1}{R} = \frac{v'}{\left[1+(v'')^2\right]^{3/2}}$

4. A beam simply supported at ends is subjected to load. The point of contraflexure will occur where :
(A) Shear force is 0 (B) Shear force is maximum
(C) Bending moment is 0 (D) Axial thrust is 0

5. If a material expands freely due to heating, it will develop
(A) Thermal stress (B) Tensile stress
(C) Bending (D) No stress

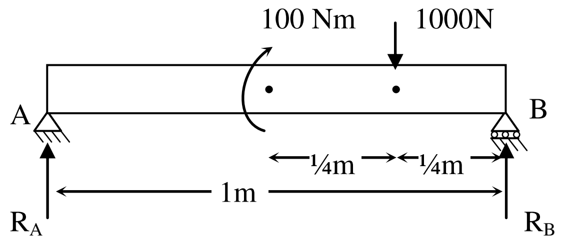
6. The stress at which extension of the material takes place more quickly as compared to the increase in load is called
(A) Elastic point of the material (B) Plastic point of the material
(C) Ultimate point of the material (D) Yielding point of the material

Q7 to Q18 carry two marks each

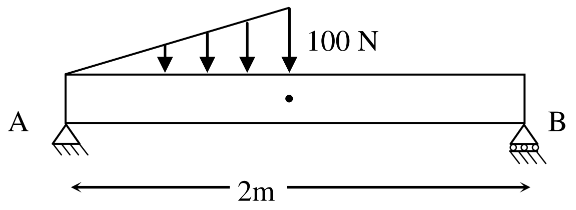
7. Two beams carrying identical loads, simply supported are having same width but beam A has double the depth compared to that of beam B. The ratio of elastic strength beam A to that of B will be:
(A) 2 (B) 4
(C) 8 (D) 1/4

8. For a loading of a beam as given below, the reaction at support A is :

(A) 100 N ↑
(B) 150 N ↑
(C) 175 N ↓
(D) 250 N ↓

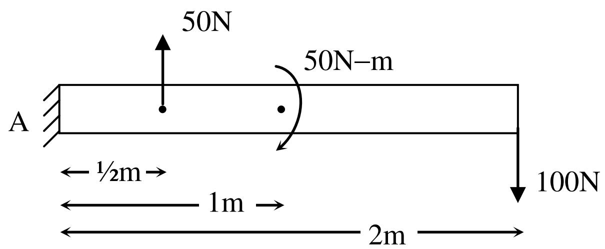


9. For loading of a simply supported beam as given below, the reaction at support B is :



(A) 33 N ↑ (B) 100 N ↑
(C) 50 N ↑ (D) 16.67 N ↑

10. For a loading of a cantilever as given below, the reaction at the fixed end is :

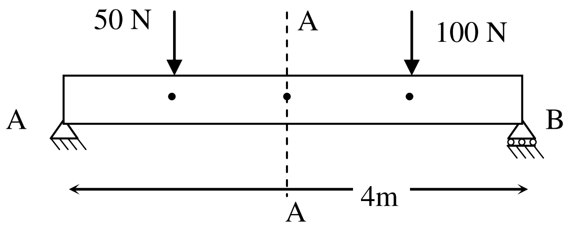


(A) 150 N ↑ (B) 175 N ↓
(C) 50 N ↑ (D) 0 N ↓

11. In prob. (8) the reaction at support B is :
(A) 800 N ↑ (B) 850 N ↑
(C) 750 N ↓ (D) 66 N ↑

12. For a given loading of a simply supported beam as shown below, the reaction at support A is :

(A) 350 N
(B) 700 N
(C) (250/4) N
(D) (350/2) N



13. In the above problem, the reaction at support B is :
(A) (250/4) N (C) 500 N
(C) (350/4) N (D) (250/8) N

14. In the above problem, the shear force at section A – A is ;
(A) (250/4) N (B) (50/4) N
(C) (350/8) N (D) 50 N

15. In the above problem, the moment at section A – A is :
(A) 175 N – m (B) 700 N – m
(C) $\left(\frac{250}{4}\right)$ N – m (D) 75 N – m

16. For a horizontal beam, at any section, σ = 6000 MPa. Cross–section is rectangular with 20mm width and 2mm height. The moment at that section will be :
(A) 60 N – m (B) 20 N – m
(C) 2 N – m (D) 8 N – m

17. A cantilever of length L is subjected to a bending moment M, at the free end. The deflection of beam at the free end is :

(A) $\dfrac{M_1L^2}{4EI}$ (B) $\dfrac{2M_1L^2}{EI}$

(C) $\dfrac{M_1L^2}{196EI}$ (D) $\dfrac{M_1L^2}{2EI}$

18. A cantilever of length L, has a bending moment M at the center. The slope θ(✰v / ✰x) at distance L/4 from the fixed end is :

(A) $\dfrac{3ML}{4EI}$ (B) $\dfrac{ML}{4EI}$

(C) $\dfrac{ML}{2EI}$ (D) $\dfrac{ML}{8EI}$

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TEST PAPER – 1

Duration : 30 Min. **Max. Marks : 25**

Q1 to Q5 carry one mark each

1. The value of Poisson's ratio for cork is taken as:
(A) Zero (B) 0.1
(C) 0.2 (D) 0.3

2. The phenomenon under which the strain of a material varies under constant stress is known as:
(A) Creep (B) Strain hardening
(C) Fatigue failure (D) Hysteresis

3. The approximate value of allowable stress for carbon steel under static loading(kg/cm^2) is:
(A) 500-1000 (B) 4000-7500
(C) 10000-15000 (D) 15000-20000

4. Within elastic limits the greatest amount of strain energy per unit volume that a material can absorb is known as:
(A) Shock proof energy (B) Resilience
(C) Proof resilience (D) Impact energy limit

5. The failure of a material under varying load, after a member of cycles of such a load is known as:
(A) Ductile failure (B) Brittle failure
(C) Impact failure (D) Fatigue failure

Q6 to Q13 carry two marks each

6. The volumetric strain of a body having modulus of elasticity E and subjected to a load P, and having Poisson's ratio 0.25 is:
(A) P/E (B) P/2E
(C) 2P/E (D) P/4E

7. In a universal testing machine during the testing of a specimen of original cross−sectional area 1 cm^2 the maximum load applied was 7.5 tonnes and neck area was 0.6 cm^2. The ultimate tensile strength of the specimen is:
(A) 12.5 tonnes/ cm^2 (B) 7.5 tonnes/ cm^2
(C) 4.5 tonnes/ cm^2 (D) 10 tonnes/ cm^2

8. In the simple bending theory, one of the assumptions usually made is that the plane sections before bending remain plane after bending. This assumption implies that:
(A) Stress is uniform in the beam cross-section
(B) Strain is uniform in the beam cross-section
(C) Stress is proportional to the distance from the neutral axis
(D) Strain is proportional to the distance from the neutral axis

9. Identify the true statement about true stress-strain method:
(A) There is no such phenomenon like true stress or true strain
(B) True stress is load per unit area (actual) and similarly true strain is determined under actual conditions
(C) This method can be used for compression tests as well
(D) It is more sensitive to changes in mechanical conditions

10. A simply supported beam of span L which carries over its full span a uniform load of W, will have zero shear force at:
(A) ends
(B) mid span
(C) both at ends as well as mid span
(D) L/4 from either ends

11. A uniform steel rope 400mts long is hung vertically. The weight of the steel is 7.8 kg/cm^2 and E = 2100 kg/mm^2 . The ratio of elongation of first 150mts to that of 300mts would be:
(A) 16:1 (B) 4:1
(C) 2:1 (D) 1:4

12. If the value of Poisson's ratio is zero, then it means that:
(A) The material is rigid
(B) The material is perfectly plastic
(C) There is no longitudinal strain in the material
(D) The longitudinal strain in the material is infinite

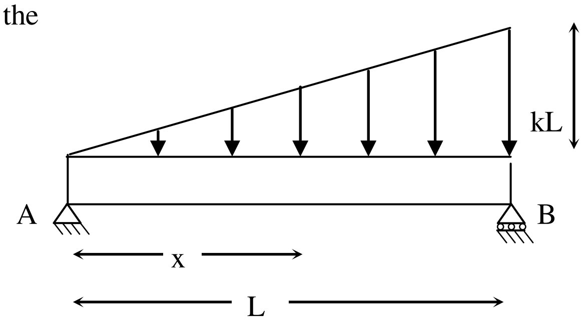
13. The number of elastic constants for a completely anisotropic elastic material which follows Hook's law is:
(A) 2 (B) 3
(C) 4 (D) 21

Q14(a) & (b) carry two marks each

Linked Answer Question

14(a). For a simply supported loading as given, the distance x at a which the shear force is 0 is

(A) L/3
(B) L
(C) $\frac{L}{\sqrt{3}}$
(D) L/2



14(b). For the above problem, the reaction at support B is :
(A) $\frac{KL^2}{2}$ (B) $\frac{KL^2}{3}$
(C) $\frac{KL^2}{6}$ (D) $\frac{KL^2}{8}$

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TEST PAPER – 2

Duration : 30 Min. **Max. Marks : 25**

Q1 to Q5 carry one mark each

1. A roller type of support is capable of giving reaction
(A) force in direction coincident with line of action
(B) force in direction perpendicular to the surface upon which the rollers rest.
(C) moment
(D) force in any direction

2. In Mohr's circle the distance of the center of the circle from y-axis is:
(A) (px – py) (B) (px + py)
(C) (px + py)/2 (D) (px – py)/2

3. In the transformation of stresses, the angle θ is taken positive:
(A) Anticlockwise (B) Clockwise
(C) Does not occur in the equation (D) Occurs but does not make a difference

4. The ratio tensile strength/shear strength should be ________ to avoid failure in shear along 45 degree planes to a material subjected to uniaxial tension:
(A) 2 (B) 1/2
(C) 1 (D) 4

5. If a cantilever beam of diameter D, cross–section A, length L is subjected to uniformly distributed load W, it will have maximum bending moment equal to :
(A) $\frac{WL^2}{2}$ (B) $\frac{WLA}{2}$
(C) $\frac{WL^2}{2A}$ (D) $\frac{WL}{2}$

Q6 to Q13 carry two marks each

6. Which material has the highest value of Poisson's ratio?
(A) steel (B) copper
(C) concrete (D) rubber

7. Identify the incorrect relationship between E, G, K and v:
(A) E = 2G(1 + v) (B) v = (E – 2G)/2G
(C) K = E(1 – 2v) (D) G = E(1 + v)/2

8. The bending moment diagram for a cantilever beam subjected to force W at the end of the beam would be :
(A) rectangle (B) parabola
(C) triangle (D) cubic Parabola

9. In the case of a cantilever beam subjected to a force W at the free end, the maximum value of bending moment is :
(A) $\frac{WL}{2}$ (B) WL
(C) $\frac{WL}{4}$ (D) $\frac{WL^2}{8}$

10. For a cantilever beam subjected to a force W at the free end, the shear force diagram looks like :

(A) Rectangle (B) Parabola
(C) Triangle (D) Trapezium

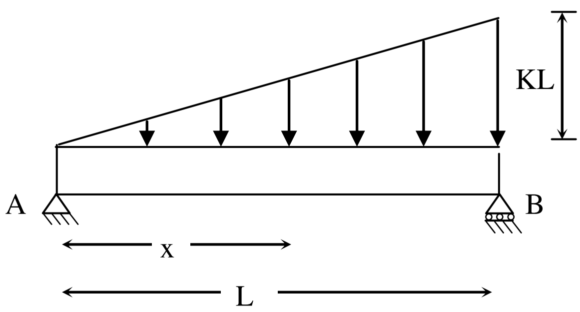
11. For the simply supported loading as shown, the maximum value of bending moment is :

(A) $-\frac{KL^3}{18\sqrt{3}}$

(B) $+\frac{KL^3}{9\sqrt{3}}$

(C) $-\frac{KL^3}{9}$

(D) $-\frac{KL^3}{18}$

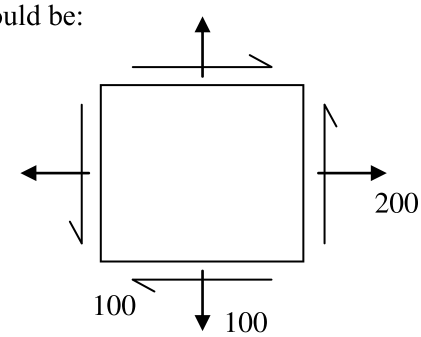


12. In the above problem, the bending moment diagram is a :

(A) Parabola (B) Trapezium
(C) Rectangle (D) Triangle

13. For the given stresses, the radius of its Mohr's circle would be:

(A) 234
(B) 145
(C) 176
(D) 112

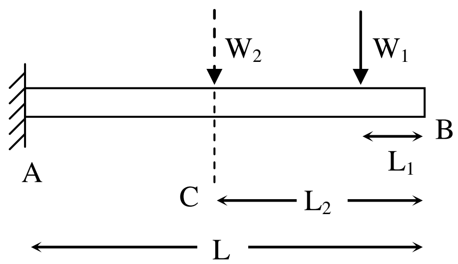


Q14(a) & (b) carry two marks each

Linked Answer Question

14(a). For the cantilever beam given below, the maximum moment will be:

(A) $W_1L_1 + W_2L_2$
(B) $W_2L_1 + W_1L_2$
(C) $W_1(L_1 - L) + W_2(L_2 - L)$
(D) $W_1(L - L_1) + W_2(L - L_2)$



14(b). In the above problem, the moment at section C – C is :

(A) W_1L_1 (B) W_2L_2
(C) $W_1(L_2 - L_1)$ (D) $W_2L_2 + W_1L_1$

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TEST PAPER – 3

Duration : 30 Min. **Max. Marks : 25**

Q1 to Q5 carry one mark each

1. A pressure vessel is said to be thin walled when the ratio of internal diameter of the vessel and wall thickness is:
(A) 5 (B) 15
(C) 10 (D) more than 20

2. If a rectangular shaft is subjected to torsion, the maximum shear stress will occur:
(A) along the diagonal (B) at the corners
(C) at the center (D) at the middle of the longer side

3. When bending moment M and torque T is applied on a shaft, then equivalent torque is:
(A) M + T (B) $\sqrt{\left(M^2 + T^2\right)}$
(C) $\frac{1}{2}\sqrt{\left(M^2 + T^2\right)}$ (D) $\frac{1}{2\left[M + \sqrt{\left(M^2 + T^2\right)}\right]}$

4. The maximum torque a circular shaft can bear is 5KNm. The bending moment it can bear when loaded with a torque value of 4KNm is:
(A) 1KNm (B) 2KNm
(C) 3KNm (D) 4KNm

5. Torsional stiffness is given by:
(A) T/ϕ (B) $T\phi$
(C) T (D) T^2/ϕ

Q6 to Q13 carry two marks each

6. In a thin spherical shell of radius r, wall thickness t when subjected to an internal pressure p, the total force normal to be diametrical plane would be:
(A) Pr/2t (B) πpr
(C) πpr^2 (D) pr/t

7. The safe working pressure for a spherical vessel 1.5 m diameter and 1.5 cm wall thickness with limiting tensile stress of 450 kg/cm^2 is:
(A) 4.5 kg/cm^2 (B) 9 kg/cm^2
(C) 18 kg/cm^2 (D) 36 kg/cm^2

8. A thin cylinder contains fluid at a pressure of 30 kg/ cm^2 and the inside radius of the shell is 60 cm. The tensile stress in the material is to be limited to 900 kg/cm^2. The shell must have minimum wall thickness of:
(A) 1 mm (B) 2.7 mm
(C) 9 mm (D) 10 mm

9. Two shafts are made of mild steel one having circular cross section and the other shaft is hollow circular with inner diameter half of the outer diameter. The ratio of the torque that can be transmitted in two cases will be:
(A) 2 (B) 17/16
(C) 1 (D) 15/16

10. A solid steel shaft of 100mm diameter, runs with a load of 100HP at $(450/\pi)$ rpm. The torque in N.m on the shaft would be around:
(A) 250 (B) 500
(C) 750 (D) 4984.6

11. Two shafts of the same material are subjected to the same torque. If the first shaft is of solid circular section and the second shaft is of hollow section whose internal diameter is $2/3^{rd}$ of the outside diameter, the ratio of weights of hollow shaft to solid shaft would be:
(A) Less than 1/2 (B) Between 0.5 and 0.99
(C) 1 (D) 1 to 1.5

12. A solid circular shaft has a T_{max} = 1000 kg/m^2 and G = 1 × 10^6 N/m^2. When subjected to torsion, it undergoes a twist of 5degrees in a length of 150cm, the radius would be:
(A) $\pi/12$ (B) $\pi/15$
(C) $54/\pi$ (D) $\pi/29$

13. A circular shaft of diameter D carries a twisting moment that develops maximum shear stress T. If a hollow shaft was used having internal diameter D/2 and D as external diameter, then the maximum shear stress would be:
(A) 1.5T (B) 1.732T
(C) 1.582T (D) 1.067T

Q14(a) & (b) carry two marks each

Linked Answer Question

14(a). A rigid beam (massless) is supported horizontally by two rods of Steel & Aluminium, 2 m & 1 m long and having cross–sectional areas as 1cm^2 and 2 cm^2 & E of 2 × 10^6 kg/cm^2 and 1 × 10^6 Kg/cm^2 respectively. What is the guideline for placing a load P on beam such that it remains horizontal.
(A) Forces on both rods should be equal
(B) Force on aluminium rod should be half the steel rod
(C) Force on aluminium rod is twice the force on steel rod
(D) Unpredictable

14(b). In above problem, determine the position of P with respect to steel rod, if beam length is 2m.
(A) 2.33 m (B) 1.33 m
(C) 4.2 m (D) 5 m

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TEST PAPER – 4

Duration : 30 Min. **Max. Marks : 25**

Q1 to Q5 carry one mark each

1. Euler's formula for a column of length ℓ, with both ends fixed is:
(A) $\pi^2/4\ EI/n\ell^2$ (B) $\pi^2/2\ EI/n\ell^2$
(C) $\pi^2\ EI/n\ell^2$ (D) $4\pi^2\ EI/n\ell^2$

2. Euler's formula, for a column of length ℓ, with one end fixed, the other end free and guided in the direction of load is:
(A) $\pi^2/4\ EI/n\ell^2$ (B) $\pi^2/2\ EI/n\ell^2$
(C) $\pi^2\ EI/n\ell^2$ (D) $2\pi^2\ EI/n\ell^2$

3. The formula for elastic strain energy for a beam in shear is :
(A) $dU = \frac{2V^2}{GA}$ (B) $dU = \frac{V^2}{4GA}$
(C) $dU = \frac{G^2}{4VA}$ (D) $dU = \frac{V^2}{2GA}$

4. The potential energy stored up in a deformed body is known as:
(A) Strain energy (B) Potential energy
(C) Resilience (D) Deformation energy

5. According to Hooke's Law, where P is the temperature stress and ΔL is the change in length :
(A) $E = (PL)\Delta L$ (B) $\Delta L = PLE$
(C) $L = \frac{P\Delta L}{E}$ (D) $\Delta L = \frac{PL}{E}$

Q6 to Q13 carry two marks each

6. For a given steel rod fixed at both ends, the $\alpha = 12 \times 10^{-6}$ /°C and E = 200 GPa. For a compressive induced stress of 24MPa, the change in temperature would be :
(A) 20°C (dec) (B) 40°C (inc)
(C) 10°C (inc) (D) 80°C (dec)

7. In an elastic material, the principal stresses at a point are 40N/mm^2, 60 N/mm^2 and 20N/mm^2. The strain energy for v = 0.5 and E = 200000N/mm^2 is:
(A) 3×10^{-4} (B) 3×10^{-5}
(C) 4×10^{-5} (D) 3×10^{-3}

8. Two rods, one of steel ($\alpha = 12 \times 10^{-6}$ /°C) and other copper ($\alpha = 2 \times 10^{-6}$ /°C) are of same length and fixed at both ends. For the same ΔT increase, the stress induced is :
(A) there is equal tension in both rods
(B) there is equal compression in both rods
(C) tension in steel rod is more than that in copper
(D) compression in steel rod is more than that in copper

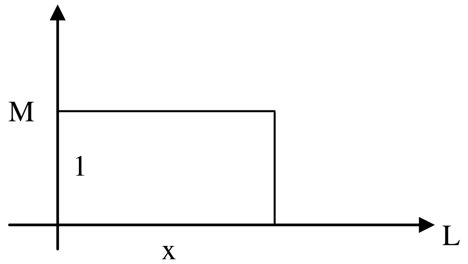
9. An axially loaded bar having length 4m, is acted upon by a load 10N. The area of cross–section is 0.01 m^2 and E = 200 GPa. The strain energy (J) is :
(A) 10^{-7} (B) 4×10^{-8}
(C) 8×10^{-7} (D) 10^{-9}

10. A simple beam is acted upon by a uniformly attributed load ω_0 N/m. Length of the beam is L, area of cross–section is A, and modulus of elasticity is E. The strain energy is :

(A) $\frac{w_0^2 L^5}{240EI}$ (B) $\frac{w_0^2 L^4}{120EI}$

(C) $\frac{w_0^2 L^5}{80EI}$ (D) $\frac{w_0^2 L^5}{60EI}$

11. For a simple beam of length L, cross–section area A, elasticity E, the strain energy for a bending moment diagram as given below is :



(A) $\frac{M^2}{2EI}$ (B) $\frac{M^2 L^2}{2EI}$

(C) $\frac{M^2 L}{2EI}$ (D) $\frac{M^2}{2LEI}$

12. The deflection of an elastic rod of constant cross–sectional area A and length L due to an axial force P applied at the free end, according to the principle of conservation of energy is :

(A) $\sqrt{2}\frac{FL}{AE}$ (B) $\frac{FL}{AE}$

(C) $\sqrt{3}\frac{FL}{AE}$ (D) $2\frac{FL}{AE}$

13. For a given plane in the complex system, the stresses are 50N/mm^2 and 40N/mm^2, and 30 N/mm^2. The maximum stress (in N/mm^2) at the elastic limit in simple tension for v = 0.25, as per strain energy theory is:

(A) 43.28 (B) 25.77

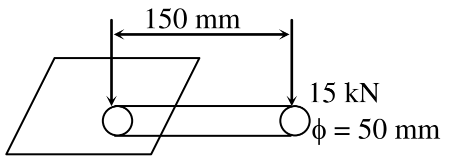
(C) 63.47 (D) 51.48

Q14(a) & (b) carry two marks each

Linked Answer Question

14(a). A circular rod of 50 mm in diameter and 150 mm long is welded to a plate by fillet welding all around the circumference as shown :

The size of the weld is 15 mm and section modulus of the weld is 22000 mm^3. What is the direct shear stress ?



(A) 9 (B) 10

(C) 12 (D) 15

14(b). In the above problem, what is max. normal stress?

(A) 11501 (B) 103.08

(C) 11049 (D) 11000

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TEST PAPER – 5

Duration : 30 Min. **Max. Marks : 25**

Q1 to Q5 carry one mark each

1. The correct formula relating moment and load per unit length is :

(A) $\frac{d^2M}{dx^2} = q$ (B) $\frac{dM}{dx} = q$

(C) $\frac{dq}{dx} = M$ (D) $\frac{d^2q}{dx^2} = M$

2. Given $q = w_0 x + w_1$ for any simply supported beam, where w_0 , w_1 are constant, the shear force at distance 2m would be :

(A) $w_0 + 2w_1$ (B) $w_0 + w_1$

(C) $w_0 + \frac{w_1}{2}$ (D) $2(w_0 + w_1)$

3. For a horizontal beam, at any section, σ = 1200 MPa, cross section of the beam is 40 mm wide and 10 mm in height. The moment at that section will be :

(A) 1200 Nm (B) 400 Nm

(C) 100 Nm (D) 800 Nm

4. A beam has σ_{max} = 800 MPa at a given section. E = 200 MPa and height of the cross–section is 10 mm. the radius of curvature would be :

(A) 200 mm (B) 800 mm

(C) 250 mm (D) 1.25 mm

5. A beam simply supported at the ends carries a load W at the centre, causing deflection δ_1. If the width of the beam is doubled the deflection at the centre under the same load will be :

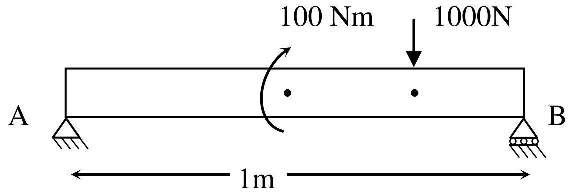
(A) δ_1 (B) $\delta_1 / 2$

(C) $\delta_1 / 4$ (D) $\delta_1 / 8$

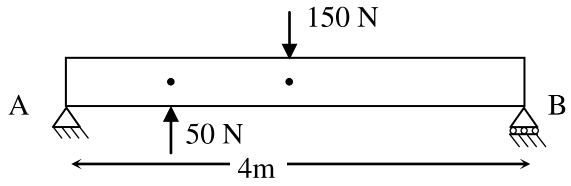
Q6 to Q13 carry two marks each

6. For a loading of a simply supported beam, the reaction at support A is 150 N ↑. The loading could be as :

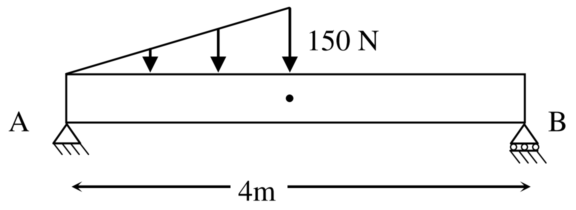
(A)



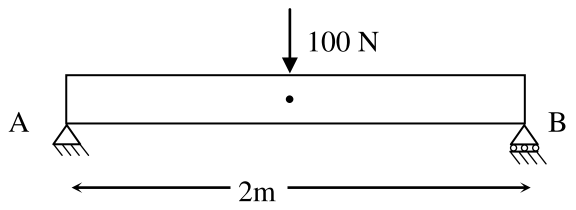
(B)



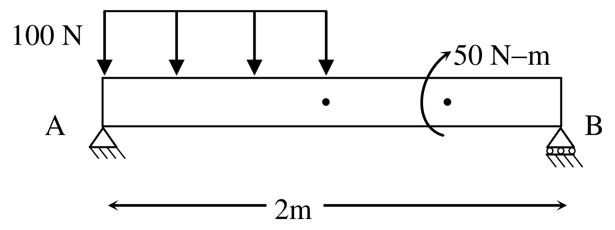
(C)



(D)

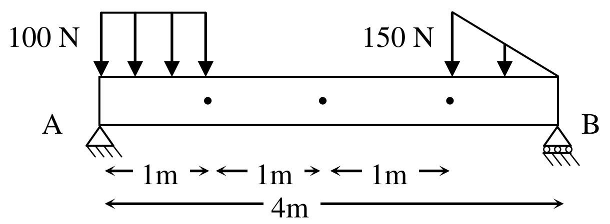


7. For a loading of a simply supported beam as given below the reaction at support A is :



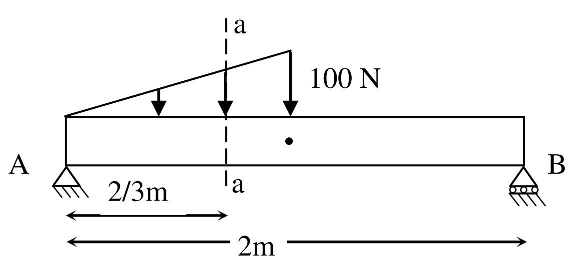
(A) 50 N ↑ (B) 80 N ↑
(C) 100 N ↓ (D) 75 N ↑

8. For a loading of a simply supported beam as given below, the reaction at support A is ;



(A) 66.67 N ↑ (B) 133.33 N ↑
(C) 100 N ↑ (D) 57 N ↑

9. For a given loading of a simply supported beam as given below, the shear force at section a – a is :



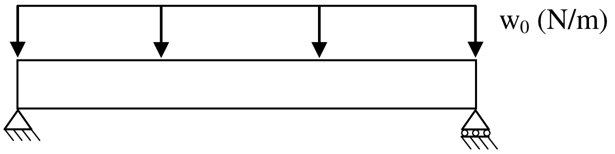
(A) 33.33 N

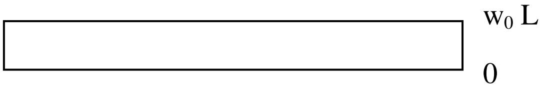
(B) $\left(\frac{100}{9}\right)$N

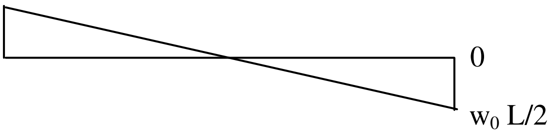
(C) $\left(\frac{200}{3}\right)$N

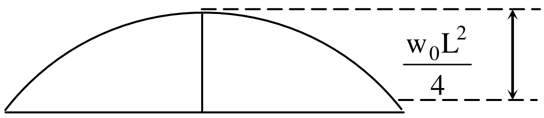
(D) $\left(\frac{200}{9}\right)$N

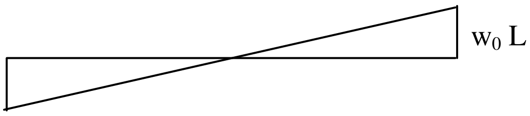
10. The shear force diagram of a simply supported beam loaded as follows looks like :



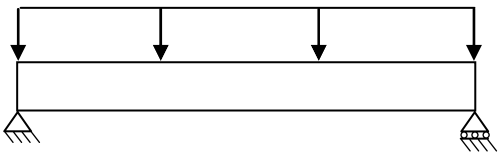
(A) 

(B) 

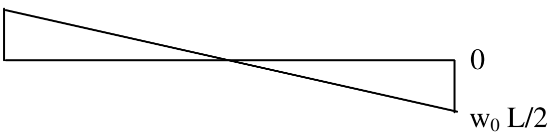
(C) 

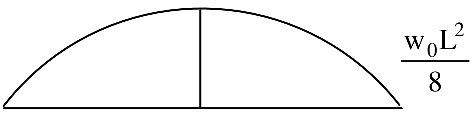
(D) 

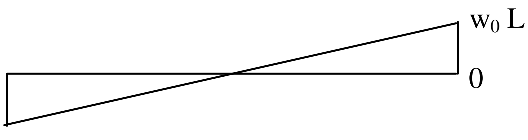
11. The moment diagram of a simply supported beam loaded as follows looks like :



(A) 

(B) 

(C) 

(D) 

12. A beam of length is hanging vertically. The deflection of the beam under its own weight if w is weight / length is :

(A) $\frac{WL^2}{AE}$ (B) $\frac{2WL^2}{AE}$

(C) $\frac{WL^2}{2AE}$ (D) $\frac{WL^2}{4AE}$

13. A cantilever is subjected to a force W at the middle of the beam. The slope at the middle of the beam is :

(A) $\frac{3WL^2}{2EI}$ (B) $\frac{WL^2}{4EI}$

(C) $\frac{9WL^2}{8EI}$ (D) $\frac{WL^2}{2EI}$

Q14(a) & (b) carry two marks each

Linked Answer Question

14(a). If a prismatic bar is subjected to direct tensile stresses σ_x and σ_y on two perpendicular faces and shear stress τ_{xy}, then stress normal to plane inclined at θ to vertical is

(A) $\frac{\sigma_x+\sigma_y}{2}+\frac{\sigma_x-\sigma_y}{2}\cos 2\theta-\tau_{xy}\sin 2\theta$

(B) $\frac{\sigma_x-\sigma_y}{2}\sin 2\theta+\tau_{xy}\cos 2\theta$

(C) $\frac{\sigma_x+\sigma_y}{2}+\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$

(D) $\frac{\sigma_x+\sigma_y}{2}-\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$

14(b). Principle stresses in above case occur at angle of

(A) $\tan^{-1}\left(-\frac{2\tau_{xy}}{\sigma_x-\sigma_y}\right)$ (B) $\frac{1}{2}\tan^{-1}\left(-\frac{2\tau_{xy}}{\sigma_x-\sigma_y}\right)$

(C) $\frac{1}{2}\tan^{-1}\left(\frac{\sigma_x-\sigma_y}{2\tau_{xy}}\right)$ (D) $\frac{1}{2}\tan^{-1}\left(-\frac{2\tau_{xy}}{\sigma_x+\sigma_y}\right)$

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PRACTICE PROBLEMS

1. A simply supported beam is subjected to a uniformly distributed force q_0 over the entire length. The maximum deflection of the beam is :

(A) $\frac{33q_0L^4}{96EI}$ (B) $\frac{5q_0L^4}{384EI}$

(C) $\frac{3q_0L^4}{8EI}$ (D) $\frac{5q_0L^4}{96EI}$

2. In an elastic material, the principal stresses at a point are 40N/mm^2, 60 N/mm^2 and 20N/mm^2. The volumetric strain for v = 0.25 and E = 200000N/mm^2 is:

(A) 2×10^{-3} (B) 4×10^{-3}

(C) 3×10^{-5} (D) 3×10^{-4}

3. When the shear strain energy in the complex system and in simple tension are equal on failure, this theory is:

(A) Rankine theory
(B) Maximum shear stress theory
(C) Strain energy theory
(D) Von–Mises theory

4. According to the maximum shear stress criterion, if the principal stresses are σ_1 and σ_2, then their relation with the yielding stress σ_y is:

(A) $\sigma_1{}^2 + \sigma_2{}^2 = \sigma_y{}^2$ (B) $\sigma_1 = \sigma_y$

(C) $\sigma_2 = \sigma_y$ (D) $\sigma_2 - \sigma_1 = \sigma_y$

5. The strain energy stored in a simply supported beam of span L and flexural rigidity EI due to a central concentrated load W, is:

(A) $\frac{W^2L^3}{48EI}$ (B) $\frac{W^2L^3}{24EI}$

(C) $\frac{W^2L^3}{96EI}$ (D) $\frac{W^2L^2}{96EI}$

6. Within elastic limits, the greatest amount of strain energy per unit volume that a material can absorb, is known as:

(A) Shock proof energy (B) Resilience

(C) Proof resilience (D) Impact energy limit

7. The modulus of resilience is:

(A) The energy stored in a body
(B) Strain energy stored in a specimen when strained within elastic limit
(C) Strain energy stored in a specimen when strained beyond elastic limit
(D) The maximum energy stored at elastic limit per unit volume

8. The strain energy of the spring when it is subjected to the greatest load which the spring can carry without suffering permanent distortion is known as:

(A) Limiting stress
(B) Proof stress
(C) Proof load stress
(D) Proof resilience

9. Given $q = w_0 x + w_1$ for any simply supported beam where w_0 and w_1 are constant, the moment at distance 2m would be :

(A) $w_0 + \frac{w_1}{2}$ (B) $\frac{w_0}{2} + \frac{4w_1}{3}$

(C) $\frac{4w_0}{3} + 2w_1$ (D) $w_0 + \frac{w_1}{2}$

10. For a simply supported beam, the moment at the ends is :

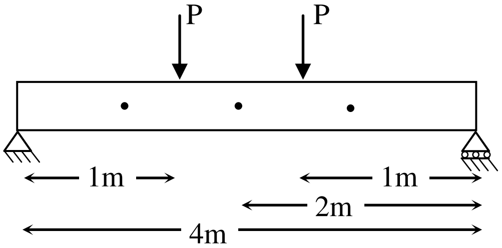
(A) zero

(B) non zero

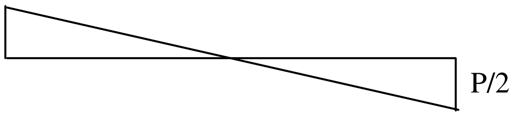
(C) equal to reaction × length of beam

(D) reaction × length / 2

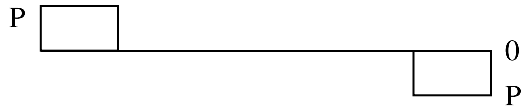
11. The shear force diagram for a simply supported beam loaded as below looks like :



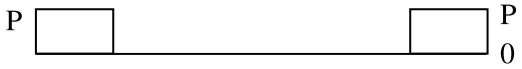
(A)



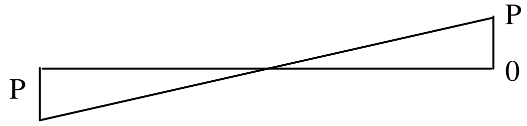
(B)



(C)



(D)



12. In the above problem, the moment diagram looks like :

(A) Trapezium (B) Rectangle

(C) Parabola (D) Square

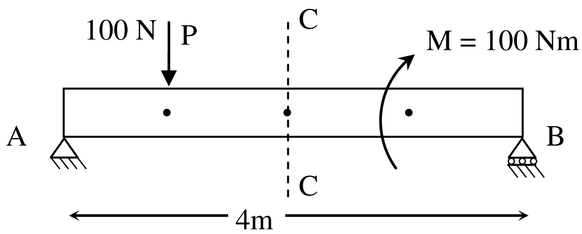
13. For a simply supported beam as loaded below, the moment at section C – C would be :

(A) 100 N – m

(B) 50 N – m

(C) 25 N – m

(D) 0 N – m



14. Which of the following is applied to brittle materials ?
(A) Maximum Principle stress theory
(B) Maximum Principle Strain theory
(C) Maximum Strain Energy theory
(D) Maximum Shear Strain theory

15. Which of the following is applied to ductile materials ?
(A) Maximum Principle stress theory
(B) Maximum Principle Strain theory
(C) Maximum Strain Energy theory
(D) Maximum distortion energy theory

16. Impact strength of a material is an index of its :
(A) hardness (B) resistance
(C) UTS (D) toughness

17. A cantilever beam of length 2m is acted upon by a force 12 N at the free end. The moment to be applied at the free end for 0 vertical deflection at that point is :
(A) 8 k N– m (B) 9 k N – m
(C) 4 k N – m (D) 16 k N – m

18. The relation between the bending moment and the moment of resistance eg. M = fz applies :
(A) Only to beams of circular cross section
(B) Only to beams of rectangular cross section
(C) When the beams are strained beyond the elastic limit
(D) When the beams are not strained beyond the elastic limit

19. A cantilever is acted upon by a force W at the free end. The ratio of maximum deflection to maximum bending stress is :
(A) $\frac{L^2}{Ed}$ (B) $\frac{3L^2}{Ed}$
(C) $\frac{2L^2}{3Ed}$ (D) $\frac{4L^2}{3Ed}$

20. A reinforced concrete beam is considered made of:
(A) Homogeneous material (B) Heterogeneous material
(C) Composite material (D) Isotropic material

21. All elastic materials:
(A) Elongate under the application of load
(B) Shrink on the application of load
(C) Permanently deform under load
(D) Do not deform under load

22. The shape of the Kern area for a rectangular section is:
(A) Circle (B) Square
(C) Rectangle (D) Parallelogram

23. A material which undergoes no deformation till its yield point is reached and then it flows at a constant stress is known as:
(A) Elasto-plastic (B) Plasto-elastic
(C) Rigid-plastic (D) Rigid-elastic

24. When a bar is lengthened or shortened by a load, it simultaneously becomes narrower or wider. The ratio of this strain in the direction of the width to the simultaneous strain in the direction of the length is called:

(A) Young's modulus (B) Hook's ratio
(C) Fick′s constant (D) Poisson's ratio

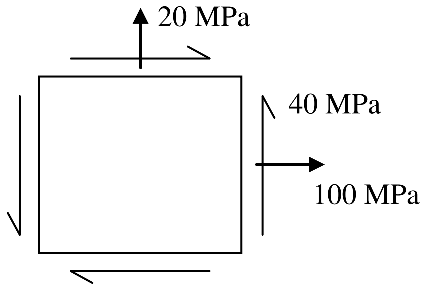
25. A dead load is:

(A) One that does not exist (B) One that is dead
(C) One that can be neglected (D) One that is constant

26. Materials exhibiting time bound behaviour are known as:

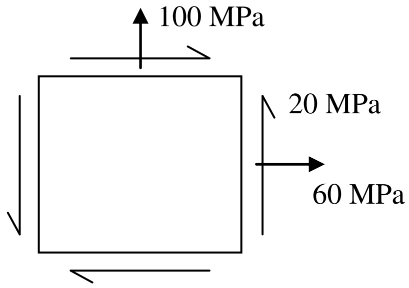
(A) Isentropic (B) Reactive
(C) Fissile (D) Visco elastic

27. For the given block, the principal stresses are :



(A) 20 MPa, – 30 MPa (B) 17.4 MPa, 83.7 MPa
(C) 116.57 MPa, –3.43 MPa (D) 50 MPa, –20 MPa

28. For the given block, the principal plane will be at an angle _________ to the current planes.



(A) 40° (B) 45°
(C) 22.5° (D) 85°

29. Consider a spherical vessel of radius 500mm having wall thickness of 5 mm. The maximum membrane stress caused by an interval pressure of 2.4 MPa would be :

(A) 500 MPa (B) 240 MPa
(C) 120 MPa (D) 480 MPa

30. A spherical shell has membrane stress equal to 50 MPa. If the pressure is doubled, radius is increased by 4 times, and thickness is halved, the final membrane stress would be :

(A) 800 MPa (B) 200 MPa
(C) 50 MPa (D) 400 MPa

31. The safe working pressure for a spherical vessel 1m in diameter and 2 cm wall thickness with limiting tensile stress of 500 kg/cm^2 is

(A) 40 kg/cm^2 (B) 70 kg/cm^2
(C) 80 kg/cm^2 (D) 20 kg/cm^2

32. A bar of length L, cross–section A and self weight W is hanging vertically. The total elongation of the bar when subjected to a load P acting axially at the bottom is :

(A) $\frac{WL}{2AE}+\frac{PL}{2AE}$ (B) $\frac{WL}{2AE}+\frac{PL}{AE}$

(C) $\frac{WL}{AE}+\frac{PL}{AE}$ (D) $\frac{2WL}{AE}+\frac{PL}{2AE}$

33. A fixed type of support is capable of ;

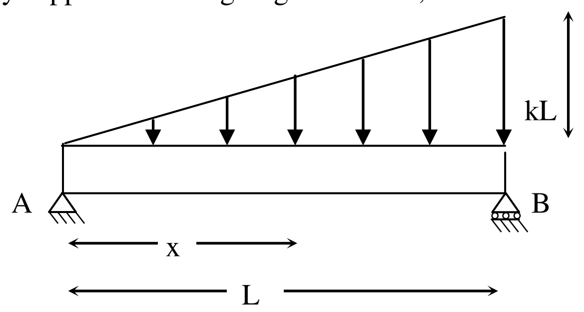
(A) Force in direction coincident with line of action

(B) Force in direction ⊥ to line of action

(C) Moment

(D) Force in any direction

34. For a simply supported loading as given below, the distance x at a which the shear force is 0 is



(A) L/3 (B) L

(C) $\frac{L}{\sqrt{3}}$ (D) L/2

35. In the above problem, the bending moment diagram is a :

(A) Parabola (B) Trapezium

(C) Rectangle (D) Triangle

36. In a beam where shear force is maximum, the bending moment will be

(A) maximum (B) zero

(C) minimum (D) No such relation between the two

37. A cantilever beam having rectangular cross–section is subjected to a load W at its free end. If the depth of the beam is doubled and load is halved, then δ_1 / δ_2 at the free end is :

(A) 1/2 (B) 1/4

(C) 1/16 (D) 2

38. A simply supported beam of length L carries a point load at its centre. The strain energy in the beam due to bending is

(A) $\frac{L^3}{96EI}$ (B) $\frac{L^3}{120EI}$

(C) $\frac{L^3}{48EI}$ (D) $\frac{L^3}{16EI}$

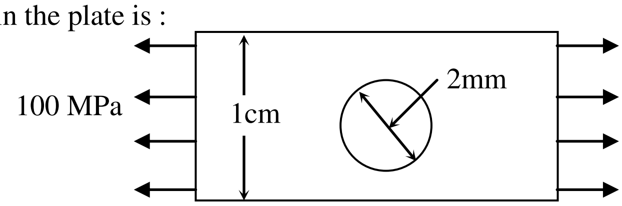
39. A rectangular plate is subjected to a uniaxial tension of 100MPa. The plate has a hole at the centre. The maximum stress in the plate is :

(A) 100 MPa

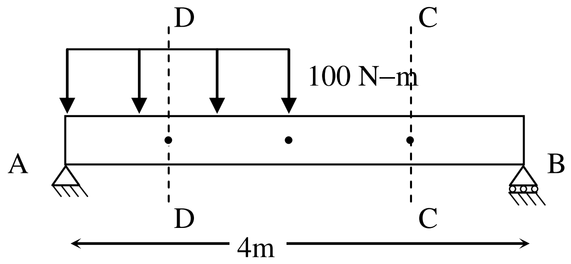
(B) 50 MPa

(C) 125 MPa

(D) 150 MPa



40. For a given loading of a simply supported beam as given below, the reaction at support A is :



(A) 250 N (B) 350 N
(C) 50 N (D) 150 N

41. For the above problem, the reaction at support B is :

(A) 250 N (B) 350 N
(C) 50 N (D) 150 N

42. In the above problem, the shear force at section C – C is :

(A) 150 N (B) 100 N
(C) 105 N (D) 50 N

43. In the above problem, moment at section C – C is

(A) 250 N – m (B) 200 N – m
(C) 350 N – m (D) 50 N – m

44. In the above problem, the shear force at section D – D is

(A) 250 N (B) 350 N
(C) 50 N (D) 150 N

45. In the above problem, the moment at section D – D is :

(A) 250 N – m (B) 350 N – m
(C) 100 N – m (D) 150 N – m

46. The correct formula relating shear force and load per unit length is :

(A) $\frac{d^2v}{dx^2} = q$ (B) $\frac{dq}{dx} = v$
(C) $\frac{d^2q}{dx^2} = v$ (D) $\frac{dv}{dx} = q$

47. A cantilever beam is subjected to a load W at its free end causing deflection δ_1. If the load is increased to 2W, causing deflection δ_2, the value δ_1 / δ_2 would be :
(A) 1 (B) 2
(C) 1/2 (D) 4

48. A cantilever beam of square cross–section is subjected to a load W at the free end. If the length of the beam is doubled and load reduced to half, the deflection at the free end as compared to original deflection would be :
(A) 2 (B) 4
(C) 8 (D) 16

49. Two beams of equal cross–section are subjected to equal bending moment. If one beam has square cross–section and other has circular cross–section, then
(A) Both beams will be equally strong
(B) Circular section beam will be economical
(C) Square sectional beam will be economical
(D) In shear, square section beam will be stronger

50. A continuous beam is :
(A) One which is constrained at both ends
(B) One resting upon several supports
(C) One which is part of a system consisting of a number of beams
(D) One which is very long as compared to width of beam

51. A beam of uniform strength is one in which :
(A) the cross–section is same throughout
(B) the bending moment is same at every section
(C) the stiffness is same at every section
(D) bending stress is same at every section

52. A beam simply supported at the ends carries a load W at the centre, causing deflection δ. If the depth of the section is doubled, the deflection at the centre will be :
(A) δ (B) $\delta / 2$
(C) $\delta / 4$ (D) $\delta_1 / 8$

53. Two beams carrying identical loads, simply supported are having same width but beam A has double the depth as compared to that of beam B. The ratio of elastic strength of beam A to that of B for same bending will be :
(A) 1/4 (B) 2
(C) 1/8 (D) 8

54. A beam simply supported carries a central load W. The ratio of maximum deflection to maximum bending stress is :
(A) $\frac{L^2}{6ED}$ (B) $\frac{L^2}{8ED}$
(C) $\frac{L^2}{48ED}$ (D) $\frac{L^2}{12ED}$

55. A material which recovers fully after unloading but not instantaneously is known as:
(A) Plastic (B) Elastic
(C) Partially elastic (D) Inelastic

56. The limit within which Hooke's law holds good is known as:
(A) Elastic limit (B) Plastic limit
(C) Yield point (D) Euler's limit

57. The bending moment diagram for a cantilever beam subjected to bending moment at the end of the beam would be:
(A) Rectangle (B) Triangle
(C) Parabola (D) Cubic parabola

58. In a beam where shear force is maximum, the bending moment will be
(A) maximum (B) zero
(C) minimum (D) No such relation between the two

59. For a given maximum shear stress the minimum diameter of solid circular shaft required to transmit H horse power at N rpm will be proportional to:
(A) (N/H) (B) $(N/H)^2$
(C) $(N/H)^3$ (D) $(N/H)^{1/3}$

60. A steel shaft 6mm in diameter turns at 10000rpm. The safe working stress in shear is 350 kg/cm^2. The maximum power that such a shaft may develop is approximately:
(A) 1524W (B) 100W
(C) 173W (D) 28W

61. For a column of length ℓ, the buckling load:
(A) Is directly proportional to the slenderness ratio
(B) Is inversely proportional to the slenderness ratio
(C) Is non-linearly proportional to the slenderness ratio
(D) Is related to the length

62. The following is the correct relationship between strain energy and load acting on an axially loaded bar :
(A) $P = \sqrt{\frac{2AEU}{L}}$ (B) $P = 2\sqrt{\frac{AE}{LU}}$
(C) $P = \sqrt{\frac{AE}{2U}}$ (D) $P = \sqrt{\frac{ALU}{2E}}$

63. A cantilever of length L is subjected to a bending moment of M_1 at the free end. The deflection at the centre of the beam is :
(A) $\frac{M_1L^2}{4EI}$ (B) $\frac{2M_1L^2}{EI}$
(C) $\frac{M_1L^2}{8EI}$ (D) $\frac{M_1L^2}{2EI}$

64. A cantilever of length L is subjected to a bending moment M at the centre. The deflection at the free end would be :
(A) $\frac{ML^2}{2EI}$ (B) $\frac{3ML^2}{8EI}$
(C) $\frac{ML^2}{48EI}$ (D) $\frac{ML^2}{9EI}$

65. A cantilever of length L, is subjected to a distributed load from L/2 < x < 1.
The magnitude of deflection of the cantilever at distance L/2 form fixed end is :
(A) $\frac{q_0L^4}{8EI}$ (B) $\frac{3q_0L^4}{48EI}$
(C) $\frac{5q_0L^4}{96EI}$ (D) $\frac{3q_0L^4}{96EI}$

66. A simply supported beam is subjected to a distributed load q_0 over the entire length. The deflection of the beam at length L/4 from fixed end is :
(A) $\frac{57q_0L^4}{6144EI}$ (B) $\frac{3q_0L^4}{48EI}$
(C) $\frac{q_0L^4}{8EI}$ (D) $\frac{3q_0L^4}{8EI}$

67. For a thin cylinder, the ratio longitudinal stress/ hoop stress is:

(A) $\frac{1}{2}$ (B) 1

(C) 2 (D) 4

68. A solid shaft has maximum shear stress of 60mpa. The diameter of the shaft for a 15hp motor operating at 20 Hz is around:

(A) 10 mm (B) 15 mm

(C) 13 mm (D) 20 mm

69. A rod of steel (E = 200 GPa and $\alpha = 12 \times 10^{-6}$ /C°) is fixed at both ends. There is no stress when the temperature is 30°C. The axial stress when the temperature is decreased to 10° C is

(A) 48 MPa (T) (B) 48 MPa (C)

(C) 24 MPa (T) (D) 24 MPa (C)

70. A copper bar having $\alpha = 2 \times 10^{-6}$ /°C and $E = 1 \times 10^{6}$ kg/cm^2 is fixed at both ends. If the temperature of the bar is decreased by 50°C then the stress induced in the bar is

(A) 100 kg/cm^2(T) (B) 200 kg/cm^2(T)

(C) 100 kg/cm^2(C) (D) 200 kg/cm^2(C)

71. A simply supported beam is subjected to a uniformly distributed force q_0 over the entire length. The slope at the centre of the beam is :

(A) $\frac{q_0L^3}{24EI}$ (B) $\frac{q_0L^3}{48EI}$

(C) $\frac{3q_0L^3}{EI}$ (D) None of the above

❑ ❑ ❑ ❑ ❑ ❑

Solutions

ANSWER KEY TO ASSIGNMENT – 1

1.	(D)	2.	(A)	3.	(D)
4.	(D)	5.	(D)	6.	(C)
7.	(B)	8.	(C)	9.	(D)
10.	(D)	11.	(A)	12.	(A)
13.	(A)	14.	(D)	15.	(D)
16.	(A)	17.	(C)	18.	(D)

ANSWER KEY TO ASSIGNMENT – 2

1.	(D)	2.	(D)	3.	(D)
4.	(D)	5.	(D)	6.	(A)
7.	(D)	8.	(C)	9.	(A)
10.	(D)	11.	(A)	12.	(C)
13.	(A)	14.	(D)	15.	(C)
16.	(B)	17.	(C)	18.	(B)

ANSWER KEY TO ASSIGNMENT – 3

1.	(B)	2.	(A)	3.	(C)
4.	(A)	5.	(D)	6.	(D)
7.	(C)	8.	(D)	9.	(B)
10.	(D)	11.	(C)	12.	(D)
13.	(C)	14.	(D)	15.	(C)
16.	(A)	17.	(C)	18.	(B)

ANSWER KEY TO ASSIGNMENT – 4

1.	(A)	2.	(C)	3.	(D)
4.	(C)	5.	(A)	6.	(A)
7.	(C)	8.	(A)	9.	(B)
10.	(A)	11.	(C)	12.	(D)
13.	(C)	14.	(D)	15.	(C)
16.	(C)	17.	(C)	18.	(C)

ANSWER KEY TO ASSIGNMENT – 5

1.	(C)	2.	(C)	3.	(B)
4.	(C)	5.	(D)	6.	(D)
7.	(B)	8.	(B)	9.	(D)
10.	(C)	11.	(B)	12.	(C)
13.	(C)	14.	(B)	15.	(D)
16.	(D)	17.	(D)	18.	(B)

❑ ❑ ❑ ❑ ❑ ❑

MODEL SOLUTIONS TO ASSIGNMENT – 1

1. (D)

2. (A)
The formula for Young's modulus is e = FL/AE, where F is the force, L is the length, A is the area and E is the Young's modulus and e is the elongation.

3. (D) **4.** (D) **5.** (D)

6. (C)

7. (B)

$$\frac{K}{G}=\frac{2(1+v)}{3(1-2v)}=\frac{2\times1.25}{3\times1/2}=5/3$$

where, K = Bulk modulus $=\dfrac{E}{3(1-2v)}$

G = Shear Modulus = $\dfrac{E}{2(1+v)}$

8. (C)

9. (D)
ℓ mm = 0.001 ℓ m. strain = change in length/ original length.

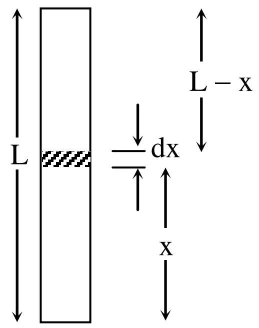
10. (D)

11. (A)
Young's modulus for any material does not depend upon whether it is being compressed or stretched. Hence, both the values are same.

12. (A) **13.** (A) **14.** (D)

15. (D)
force = shear stress × area
= 30 × π × D × L = 36.5 tonnes approx.

16. (A)
We solve this problem using integration, as the force due to the weight is not a point force but a uniformly distributed force acting over the length.



Taking an element of length dx at distance x from bottom the deflection δd is

$$\delta d = \frac{Pdx}{AE} \text{ where } P = wx$$

$$\therefore \quad \Delta = \int \delta d = \int_0^L \frac{wxdx}{AE} = \left.\frac{wx^2}{2AE}\right|_0^L$$

$$= \frac{wL^2}{2AE} = \frac{WL}{2AE}$$

17. (C)

18. (D)

❑ ❑ ❑ ❑ ❑ ❑

MODEL SOLUTIONS TO ASSIGNMENT – 2

1. (D)

2. (D)

Now, $\frac{dM}{dx} = V = 0$

$\therefore$ As $\frac{dM}{dx} = 0$, M is maximum or minimum at that point.

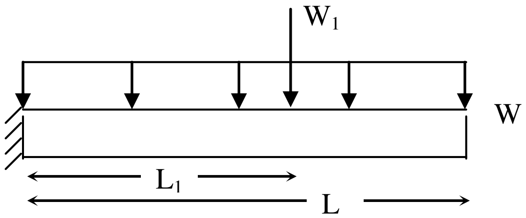
3. (D)

4. (D)

5. (D)

6. (A)

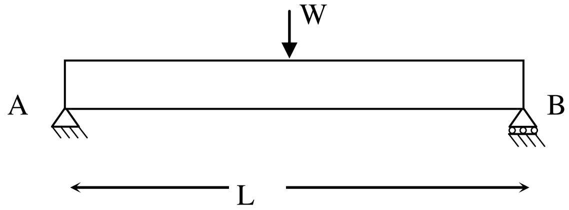
7. (D)



Taking a section at a distance of y from the wall and writing all the other forces acting at their respective distances on a free body diagram, we do a moment balance of the situation. The maximum bending moment occurs at the free end and is given as:

$\therefore$ $M = WL/2 + W_1(L - L_1)$

8. (C)



We first find out the reaction forces acting at the two ends. This will help in computing the bending moment, because the external load is itself acting at the center and hence cannot be taken into account.

The reaction force = W/2 at each end.

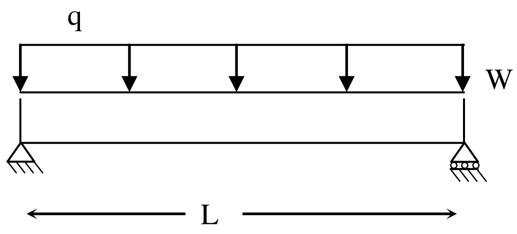
Now, taking a section at a distance of y from the wall and writing all the other forces acting at their respective distances on a free body diagram, we do a moment balance of the situation. We get the maximum bending moment at the centre as the maximum.

$\therefore$ $M = W/2 \times L/2 = WL/4$

9. (A)

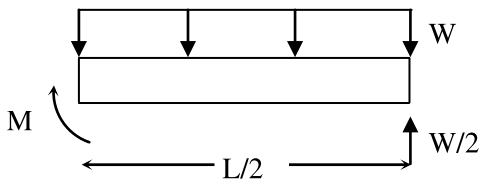
From Euler's formula, M = EI/R. Here, $I = bh^3/12$. The only term varying here is b, i.e. width. Hence as we can see that they are directly proportional, answer is (A).

10. (D)



$q = (W/L)$

Taking a cut at the centre and finding the moment there,

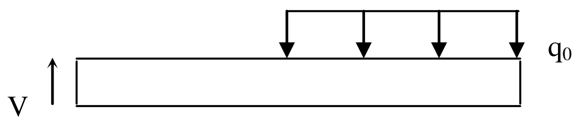


$\therefore \quad M + \frac{W}{2} \times \frac{L}{4} = \frac{W}{2} \times \frac{L}{2}$

$M = \frac{WL}{8}$

11. (A)

Taking section at fixed end



$\therefore$ Doing vertical force balance,

$V = \frac{q_0 L}{2}$

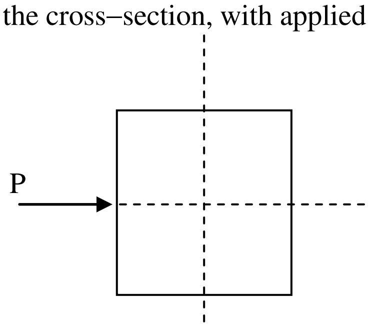
12. (C)

From 0 to L/2, $V = q_0 x$

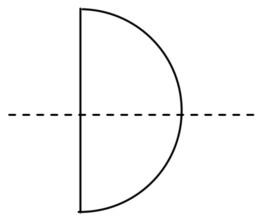
After L/2 till the fixed end, $V = \frac{q_0 L}{2}$ (cons)

13. (A)

Taking the cross–section, with applied load P,



The shear stress distribution diagram looks like,



Hence, parabola (A).

14. (D)

$\sigma_x = 50$

$\sigma_y = 100$

$\tau = 100$

$\therefore$ Principle stress

$$= \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= 75 \pm \sqrt{25^2 + 100^2}$$

$= 75 \pm 103.07$

$= 178.07$ MPa or -28.07 MPa

$\therefore$ Maximum Principle Stress $= \dfrac{\sigma_1 - \sigma_2}{2}$

$= 103.07$ MPa

15. (C)

$$\sigma_x{}' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

Now, $\sigma_x{}' = \dfrac{3+1}{2} + \dfrac{3-1}{2}\cos(-45°) + 2\sin(-45°)$

$= 1.29$ MPa

16. (B)

Now,

$$\tau_{xy} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{3-1}{2}\right)\sin(-45°) + 2\cos(-45°)$$

$= 1 \times 0.707 + 2 \times 0.707 = 2.12$ MPa

17. (C)

18. (B)

$$\tau_{xy} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \sqrt{\left(\frac{3-1}{2}\right)^2 + 2^2} = 2.24\,\text{MPa}$$

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MODEL SOLUTIONS TO ASSIGNMENT – 3

1. (B)
As a sphere is symmetrical, the longitudinal and hoop stress is the same. Hence, the ratio is 1.

2. (A)

3. (C)

4. (A)
J in this equation is the polar moment of inertia, which is the above value for a circular solid shaft of diameter D.

5. (D)

6. (D)

7. (C)
Hoop stress is given by: $\sigma_1 = \frac{pr}{t} = 0.8 \times \frac{1}{(10 \times 10^{-3})} = 80 \text{ MPa}$

Cylindrical stress is given by: $\sigma_2 = \frac{pr}{2t} = 0.8 \times \frac{1}{(2 \times 10 \times 10^{-3})} = 40 \text{ MPa}$

8. (D)
$\varepsilon_1 = (\sigma_1 - v\sigma_2) / E = 0.35 \times 10^{-3}$ mm/mm
Now, $\varepsilon_1 = [2\pi(r + d) - 2\pi r] / 2\pi r = d/r$
Hence, $d = r \times \varepsilon_1 = 0.35$ mm

9. (B)
$\sigma_1 = \sigma_2 = pr / 2t = 40$ MPa

10. (D)
$\varepsilon_1 = (\sigma_1 - v\sigma_2) / E = 0.15 \times 10^{-3}$ mm/mm
Now, $\varepsilon_1 = [2\pi(r + d) - 2\pi r] / 2\pi r = d/r$
Hence, $d = r \times \varepsilon_1 = 0.35$ mm

11. (C)
For the circular section, stress1 = $P / \pi D^2 / 4$
For the hollow section, stress 2 = $\frac{P}{\left\{\frac{\pi D^2}{4} - \frac{\pi D^2}{16}\right\}}$
Hence, stress1 / stress2 = 3/4

12. (D)
For the circular section, stress1 = $P / \pi D^2 / 4$
For the hollow section, stress2 = $P / \{ \pi D^2 /4 - \pi D^2 /36\}$
Hence, stress1 / stress2 = 8/9

13. (C)

$J = \pi d^4/32 = 402\ mm^4$

$$\tau_{max} = \frac{T.R_o}{J} = \frac{40\times10^3\times4}{402} = 400\ \text{MPa}$$

14. (D)

$I_p = \pi(D_o^4 - D_i^4)/32 = 9274\ mm^4$

$$\tau_{max} = \frac{T.R_o}{J} = \frac{40\times10^3\times10}{9720} = 43.1\ \text{MPa}$$

15. (C)

As calculated above, $J = 9274\ mm^4$

$$\tau_{min} = \frac{T.R_i}{J} = \frac{40\times10^3\times8}{9720} = 34.5\,\text{MPa}$$

16. (A)

$$T = 159\times\frac{KW}{f} = \frac{159\times150}{0.3} = 79500\ \text{N–m}$$

Now, $\frac{J}{R} = \frac{T}{\tau_{max}} = 1.14\times10^6\,mm^3$

$\frac{J}{R} = \frac{\pi R^3}{2}$, therefore, R = 90 mm

Diameter is around 180 mm

17. (C)

$$T = 159\times\frac{KW}{f} = \frac{159\times150}{300} = 79.5\,\text{N–m}$$

Now, $\frac{J}{R} = \frac{T}{\tau_{max}} = 5.81\times10^6\,mm^3$

$\frac{I_P}{c} = \frac{\pi c^3}{2}$ $\quad\therefore$ c = 9 mm.

Diameter is around 18 mm.

18. (B)

Shear stress = $Tr/J = 16T/\pi D^3$

Bending stress = $Mr/J = 32T/\pi D^3$

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MODEL SOLUTIONS TO ASSIGNMENT – 4

1. (A)

2. (C)

3. (D)
When both ends are fixed, $W = 4\pi^2 EL/n\ell^2$, which is the maximum.

4. (C)

5. (A)

6. (A)

7. (C)
On increasing the temperature the body expands and hence compressive forces act on it.
Now, stress = α t E
$= 12\times10^{-6}\times50\times200 = 120\text{MPa(C)}$

8. (A)
$\delta\ell = L \times \alpha \times \Delta t = 0.005.$
Supports yield by 0.01 cm. Hence, stress induced is zero.

9. (B)
For the steel rod,
Stress $= \alpha\ E\ \Delta\ T$
$= 12 \times 10^{-6} \times 30 = 72$ MPa (C)

10. (A)
Now, $\Delta T = \dfrac{\text{Stress}}{\alpha E} = \dfrac{100}{1\times10^6\times2\times10^{-6}} = 50°C$

The temperature would be an increase at the stress is compressive.

11. (C)
Since free at one end, hence no stress can develop free expansion.

12. (D)
For any axially loaded bar,
$$U = \frac{P^2L}{2AE}$$
$$\Rightarrow \quad U = \frac{100\times1}{2\times0.1\times2\times10^9} = 2.5\times10^{-9}\text{ J}$$

13. (C)
For an axially loaded bar,
$$U = \frac{P^2L}{2AE}, \quad \text{Now, } \Delta = \frac{PL}{AE}$$
$$\therefore \quad U = \frac{AE\Delta^2}{2L}$$

$\Rightarrow \quad \sqrt{\dfrac{2LU}{AE}} = \Delta = \sqrt{\dfrac{2\times1\times10\times10^{-9}}{0.01\times20\times10^{9}}}$

$= 10^{-8}$ m

14. (D)

15. (C)

For a beam in bending

$$U = \int_L \frac{M^2}{2EI}dx$$

$$= \int_0^L \frac{P^2(L-x)^2}{2EI}dx$$

$$= \frac{P^2L^3}{2EI} - \frac{P^2L^3}{2EI} + \frac{P^2L^3}{6EI} = \frac{P^2L^3}{6EI}$$

16. (C)

17. (C)

For a volume,

$$U = \frac{1}{2E}\left[\sigma_1{}^2 + \sigma_2{}^2 + \sigma_3{}^2 - 2v(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)\right]$$

$= 221 \times 10^{-4}$ per unit volume.

18. (C)

$$\sigma = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

$$= \frac{1}{2}(40+20) + \frac{1}{2}\sqrt{(40-20)^2 + 4\times60^2}$$

$= 90.83$ N/mm^2

≈ 91 N/mm^2

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MODEL SOLUTIONS TO ASSIGNMENT – 5

1. (C)

2. (C)

Now, we know that, $\sigma = -\frac{Ey}{R}$

$\Rightarrow$ $R = -\frac{200\times10^9\times1\times10^{-3}}{300\times10^6} = \left(\frac{2}{3}\right)m = 666.6mm$

3. (B)

4. (C)

5. (D)

6. (D)

7. (B)

We know that, M/y = E/I.

Therefore, M = Ey/I = constant for both beams as they are under the same load.

$I = by^3/12$. Hence E is proportional to the square of the depth.

8. (B)

R_A, R_B are the reactions.

Doing moment balance about support B,

$$R_A \times 1 + 100 = 1000 \times \frac{1}{4}$$

$\Rightarrow$ $R_A = +250 - 100 = 150$ N ↑

9. (D)

R_B is the reaction force at support B

Taking moment about support A,

$$R_B \times 2 = \int_0^1 100x \cdot xdx$$

$$R_B = \frac{100\cdot1}{3\times2} = 16.67 \text{ N}$$

10. (C)

We do only a vertical force balance to find the reaction

$\therefore$ $R_A + 50 = 100$

$\Rightarrow$ $R_A = 50$ N

11. (B)

As calculated before, the reaction at support A is

$R_A = 150$ N ↑

$\therefore$ $R_B = 1000 - 150 = 850$ N ↑

12. (C)

We first find the reactions at the supports

Taking moment about B,

$R_A \times 4 = 50 \times 3 + 100 \times 1$
$= 250$
$\Rightarrow R_A = (250/4)$ N

13. (C)
We know $R_A = (250/4)$ N
Doing vertical force balance

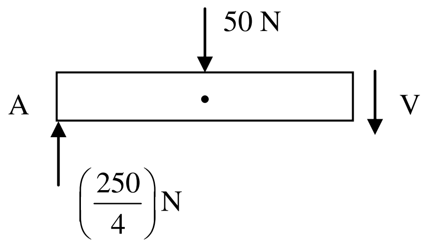
$$50 + 100 = \frac{250}{4} + R_B$$

$$\Rightarrow R_B = \left(\frac{350}{4}\right) \text{N}$$

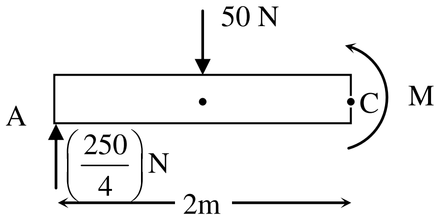
14. (B)
Doing a vertical force balance,

$$V + 50 = \frac{250}{4}$$

$$\Rightarrow V = \left(\frac{50}{4}\right) \text{N}$$



15. (D)



$$\frac{250}{4} \times 2 = 50 \times 1 + M$$

$\Rightarrow M = 75$ N – m

16. (D)
According to Euler's law,

$$\frac{M}{I} = \frac{\sigma}{y}$$

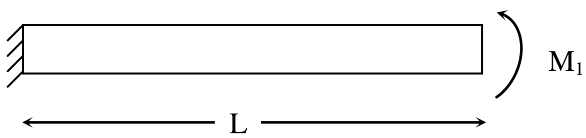
$$\Rightarrow M = -\frac{\sigma I}{y} = -\frac{\sigma \cdot bh^3}{12 \cdot h/2}$$

$$= -\frac{\sigma bh^2}{6}$$

$$= -\frac{600 \times 10^6 \times 20 \times 2^2 \times 10^{-9}}{6}$$

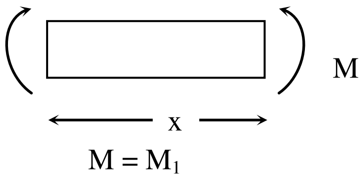
$= -8$ Nm

17. (D)



The bending moment at any section C – C is

Now, $\dfrac{\partial^2 y}{\partial^2 x} = \dfrac{M}{EI}$



$\Rightarrow \dfrac{\partial y}{\partial x} = M_1 x + C_1$

Now, $\dfrac{\partial y}{\partial x}(x = 0) = 0$

$\therefore C_1 = 0$

$y = \dfrac{M_1 x^2}{2EI} + C_2$

y = 0 at x = 0 $\Rightarrow$ $C_2 = 0$

$\therefore y = \dfrac{M_1 L^2}{2EI}$

18. (B)

Now from 0 < x < L/2

$\dfrac{\partial^2 y}{\partial x^2} = \dfrac{M}{EI}$

$\Rightarrow \dfrac{\partial y}{\partial x} = \dfrac{M}{EI} x + C_1$

Now, C_1 as $\left.\dfrac{\partial y}{\partial x}\right|_{x=0} = 0$

$\therefore \theta(x) = \dfrac{M}{EI} x \quad \Rightarrow \left.\theta\right|_{x=L/4} = \dfrac{ML}{4EI}$

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ANSWER KEY TO TEST PAPER – 1

1.	(A)	2.	(A)	3.	(B)
4.	(C)	5.	(D)	6.	(B)
7.	(C)	8.	(D)	9.	(A)
10.	(B)	11.	(D)	12.	(A)
13.	(D)	14(a).	(C)	14(b).	(B)

ANSWER KEY TO TEST PAPER – 2

1.	(A)	2.	(C)	3.	(A)
4.	(A)	5.	(D)	6.	(D)
7.	(A)	8.	(C)	9.	(B)
10.	(A)	11.	(B)	12.	(A)
13.	(D)	14(a).	(D)	14(b).	(C)

ANSWER KEY TO TEST PAPER – 3

1.	(D)	2.	(D)	3.	(B)
4.	(C)	5.	(A)	6.	(C)
7.	(C)	8.	(D)	9.	(D)
10.	(D)	11.	(B)	12.	(C)
13.	(D)	14(a).	(C)	14(b).	(B)

ANSWER KEY TO TEST PAPER – 4

1.	(D)	2.	(A)	3.	(D)
4.	(C)	5.	(D)	6.	(C)
7.	(D)	8.	(D)	9.	(A)
10.	(A)	11.	(C)	12.	(B)
13.	(D)	14(a).	(A)	14(b).	(B)

ANSWER KEY TO TEST PAPER – 5

1.	(A)	2.	(D)	3.	(D)
4.	(D)	5.	(B)	6.	(A)
7.	(A)	8.	(C)	9.	(B)
10.	(B)	11.	(C)	12.	(C)
13.	(B)	14(a).	(A)	14(b).	(A)

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MODEL SOLUTIONS TO TEST PAPER – 1

1. (A) **2.** (A) **3.** (B)

4. (C) **5.** (D)

6. (B)

Volumetric strain = $\frac{P}{E} \times \left(\frac{1-2}{m}\right)$.

Now, 1/m = 0.25.

Hence, strain = P/2E

7. (C)

Ultimate tensile strength = maximum applicable load/ cross–sectional area

Hence, UTS = 7.5 / 1 = 7.5 tonnes/ cm^2

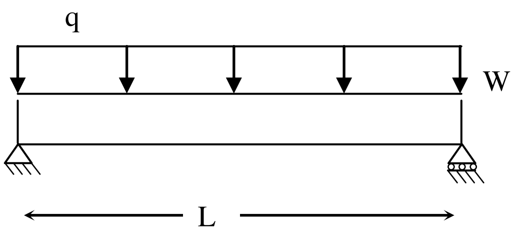
8. (D)

As the plane remains straight and does not bend, the length remains same at that plane. Hence, strain is zero there, and is proportional to the distance from the neutral axis.

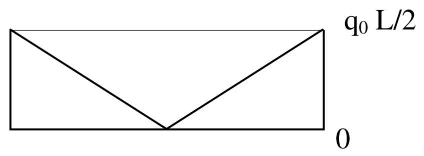
9. (A)

10. (B)

For the beam loading as shown,



The shear force diagram is as follows:



Hence, we can see that the shear force is 0 at the centre.

11. (D)

y = FL/AE. Now, A and E are constants, but F is the weight, which is double for 300mts and also the length. Hence, ratio is 1 : 4.

12. (A)

13. (D)

14(a). (C)

Now, the point of 0 shear force occurs where the reaction on the left is balanced by the applied load

∴ First finding the reaction at support A,

$$R_A \times L = \frac{L}{2} \times KL \times \frac{L}{3} = \frac{KL^2}{6}$$

$$\Rightarrow \quad R_A = \frac{KL^2}{6}$$

$\therefore$ Now, at any distance x,

$$\frac{KL^2}{6} = \frac{1}{2} \times Kx \times x$$

$$\Rightarrow \quad x = \frac{L}{\sqrt{3}}$$

14(b). (B)

Taking moment about point A,

$$R_B \times L = \frac{1}{2} \times L \times KL \times \frac{2L}{3}$$

$$\Rightarrow \quad R_B = \frac{KL^2}{3}$$

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MODEL SOLUTIONS TO TEST PAPER – 2

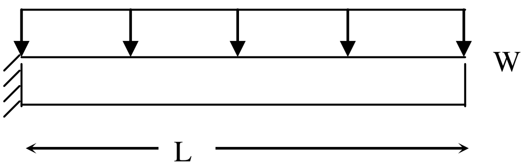
1. (A)

2. (C)

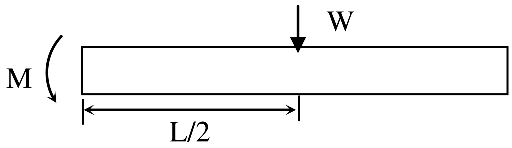
3. (A)

4. (A)

5. (D)



The maximum bending moment would take place at the fixed end.



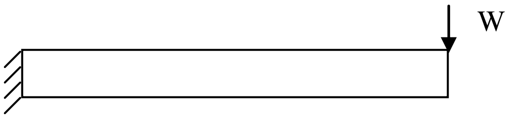
The distributed load can be substituted by a single load W, acting at the centre of the beam.

$\therefore \quad M = \frac{WL}{2}$

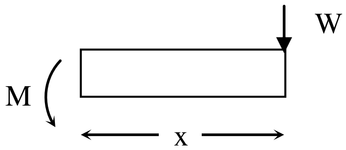
6. (D)

7. (A)

8. (C)



Taking any section at distance x,

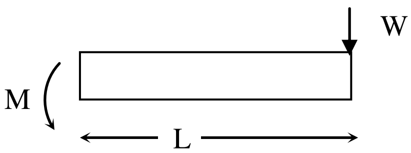


$\therefore$ M = Wx

$\therefore$ The bending moment diagram is like a triangle.

9. (B)

The maximum bending moment takes place at the fixed end

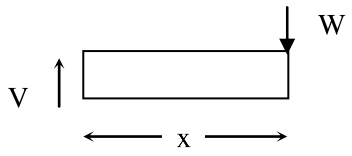


Doing moment balance

M = WL

10. (A)

Taking cut at any section distance from free end and doing vertical force balance,

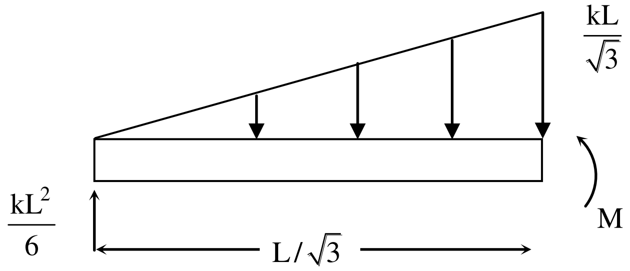


$\therefore$ V is not a function of x and = W

$\therefore$ Rectangle

11. (B)

Finding the moment at a distance $L\sqrt{3}$ from



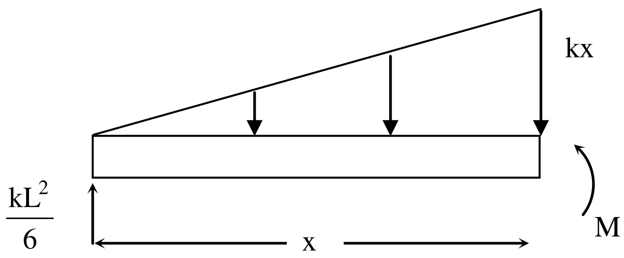
$$M + \frac{KL}{\sqrt{3}} \times \frac{1}{2} \times \frac{L}{\sqrt{3}} \times \frac{L}{3\sqrt{3}} = \frac{KL^2}{6} \times \frac{L}{\sqrt{3}}$$

$$M = \frac{KL^3}{6\sqrt{3}} - \frac{KL^3}{18\sqrt{3}} = -\frac{KL^3}{9\sqrt{3}}$$

(The reason for taking x = $\frac{L}{\sqrt{3}}$ is because shear force 0 at $\frac{L}{\sqrt{3}}$ and we know that moment is maximum or minimum when V = 0)

12. (A)

At any distance x,



$$M + Kx \times \frac{1}{2} \times x \times \frac{1}{3} x = \frac{KL^2}{6} \times x$$

$$M = \frac{Kx}{6}\left(L^2 - x^2\right)$$

$\therefore$ It is a parabola.

13. (D)

τ_{max} = radius

$$= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$

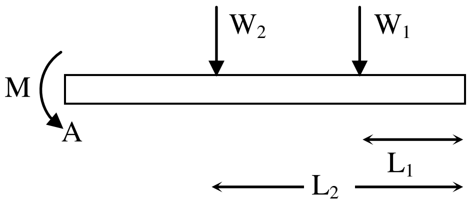
$= \sqrt{\left(\frac{200-100}{2}\right)^2 + 100^2}$

= 112 MPa

$\therefore$ Radius = 112

14(a). (D)

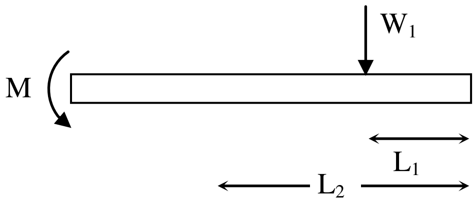
Moment will be maximum of the support end (A).



$\therefore \quad M = W_2(L-L_2)+W_1(L-L_1)$

14(b). (C)

Taking the free body diagram, to the right of section C – C.



$\therefore \quad M = W_1(L_2-L_1)$

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MODEL SOLUTIONS TO TEST PAPER – 3

1. (D)

2. (D)

We know that $T_{max} = T_c/I_p$. As we see it is directly proportional to the distance, hence, it will be maximum at the longer side.

3. (B)

4. (C)

$\sqrt{\left(M^2 + 4^2\right)} = 5$

M = 3 K–Nm

5. (A)

6. (C)

The internal pressure p is multiplied by the projected area πr^2 and this gives the total force acting normal to the diametrical plane.

7. (C)

stress = $\frac{pr}{2t}$, hence, p = $\frac{450 \times 2 \times 1.5 \times 2}{1500}$ = 18 kg/cm^2

8. (D)

t = pr/2 × stress = 30 × 60/ 2 × 900 = 1cm = 10 mm

9. (D)

The formula for calculating torque is directly proportional to the polar moment of inertia, keeping the other parameters i.e. external diameter D and maximum allowable stress constant.

Now, for the solid shaft, $J = \pi D^4/32$

And for the hollow shaft $J = \pi D^4/32 - \pi(D/2)^4/32 = (15/16)\pi D^4/32$

10. (D)

$T = 119 \times hp/f = \frac{119 \times 100}{\frac{450}{60\pi}} = 4984.6$ N.m

11. (B)

The ratio of weights will be as the ratio of polar moments of inertia.

Now, for the solid shaft, $J = \pi D^4/32$.

And for the hollow shaft $J = \pi D^4/32 - \pi(2D/3)^4/32 = (5/9)\pi D^4/32$

Therefore, ratio is 5/9 > 0.5

12. (C)

$$\frac{\tau}{R} = \frac{G\theta}{\ell}$$

$$\frac{1000}{R} = 1 \times 10^6 \times \frac{5\pi}{180} \times \frac{1}{1.5} = \frac{54}{\pi}$$

13. (D)

The formula for calculating torque is directly proportional to the polar moment of inertia, keeping the other parameters i.e. external diameter D and maximum allowable stress constant.

Now, for the solid shaft, $J = \pi D^4/32$
And for the hollow shaft $J = \pi D^4/32 - \pi(D/2)^4/32 = (15/16)\pi D^4/32$
Therefore, ratio = 16/15 = 1.067

14(a). (C)
If both stretch horizontally, beams will remain horizontal.

$$\left(\frac{FL}{EA}\right)_{Steel} = \left(\frac{FL}{EA}\right)_{Aluminium}$$

$$\Rightarrow \frac{F_S}{F_A} = \frac{1}{2} \times \frac{2\times10^6}{1\times10^6} \times \frac{1}{2} = \frac{1}{2}$$

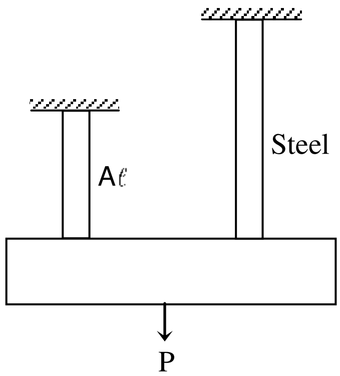
14(b). (B)
For static equilibrium, $\sum F_Y = 0$ & $\sum M_S = 0$

$$F_S + F_A = P$$

$$\frac{1}{2}F_A + F_A = P$$

$$\Rightarrow F_A = \frac{2P}{3} \quad \& \quad F\times\ell = \frac{2P}{3}\times2$$

$$\Rightarrow \ell = \frac{4}{3} = 1.33 \text{ m}$$



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MODEL SOLUTIONS TO TEST PAPER – 4

1. (D)

2. (A)

3. (D)

4. (C)

5. (D)

6. (C)

Now, $\Delta T = \dfrac{\text{Stress}}{\alpha E} = \dfrac{24 \times 10^6}{12 \times 10^{-6} \times 200 \times 10^9} = 10°C$

As the stress is compressive, the temperature change is an increment.

7. (D)

For a volume,

$$U = \frac{1}{2E}\left[\sigma_1{}^2 + \sigma_2{}^2 + \sigma_3{}^2 - 2v\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3\right)\right]$$

$$= \frac{1}{2 \times 2 \times 10^5}\left[(16+36+4)\times 100 - \frac{2}{2}(24+12+8)\times 100\right]$$

$= 3 \times 10^{-3}$ per unit volume

8. (D)

9. (A)

$U = \dfrac{P^2L}{2AE}$ for an axially loaded bar

$\therefore \quad U = \dfrac{100 \times 4}{2 \times 0.01 \times 200 \times 10^9} = 10^{-7}$ J

10. (A)

Now, at any length x,

$$M = \frac{w_0Lx}{2} - \frac{w_0x^2}{2}$$

$$M^2 = \frac{w_0{}^L L^2x^2}{4} + \frac{w_0{}^2x^4}{4} - \frac{w_0{}^2Lx^3}{2}$$

$$U = \int_L \frac{M^2}{2EI}dx$$

$$= \frac{1}{2EI}\left[\frac{w_0{}^2L^5}{12} + \frac{w_0{}^2L^5}{20} - \frac{w_0{}^2L^5}{8}\right]$$

$$= \frac{w_0{}^2L^5}{2EI}\left[\frac{10+6-15}{120}\right]$$

$$= \frac{w_0{}^2L^5}{240EI}$$

11. (C)

For a beam,

$$U = \int_L \frac{M^2}{2EI}dx = \frac{M^2L}{2EI}$$

12. (B)

$$U = \frac{\sigma^2}{2E}\cdot AL = \frac{P^2L}{2AE}$$

From W = U

$$\frac{P\Delta}{2} = \frac{P^2L}{2AE}$$

$$\Rightarrow \Delta = \frac{FL}{AE}$$

13. (D)

According to Strain Energy Theory

$$\left(\sigma_1{}^2 + \sigma_2{}^2 + \sigma_3{}^2\right) - 2v\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3\right) = \sigma^2$$

$$\Rightarrow \quad \sigma^2 = \left(50^2 + 40^2 + 30^2\right) - 2\times 0.25\left[2000 + 1200 + 1500\right]$$

$$= 5000 - \frac{4700}{2} = 2650$$

$$\Rightarrow \quad \sigma = 51.48 \text{ N/mm}^2$$

14(a). (A)

d = 50 mm, e = 150 mm, P = 15 × 1000 N

Z = 22000 mm^3, t = throat thickness

The throat area for the circular fillet weld,

$$A = t \times \pi D$$
$$= 0.707 \times S\pi D$$
$$= 0.707 \times 15 \times 3.14 \times 50$$
$$= 1664.9 \text{ mm}^2$$

Direct shear stress = $\tau = \frac{P}{A} = \frac{15000}{1664.9} = 9$

14(b). (B)

Given Z = 22000 mm^3

Bending stress, $\sigma_b = \frac{M}{Z} = \frac{P\times e}{Z}$

$$= \frac{15000\times 150}{22000} = 102.3$$

Max. normal stress $\sigma_{t\,max} = \frac{\sigma_b}{2} + \frac{1}{2}\sqrt{\left(\sigma_b\right)^2 + 4\tau^2}$

$$= \frac{102.3}{2} + \frac{1}{2}\sqrt{(102.3)^2 + 4(9)^2}$$

$$= 103.08$$

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MODEL SOLUTIONS TO TEST PAPER – 5

1. (A)

2. (D)

$$\frac{dF}{dx} = q$$

$$\Rightarrow \; F = \int q dx = \frac{w_0 x^2}{2} + w_1 x$$

$$= 2w_0 + 2w_1 \qquad \text{at } x = 2m$$

3. (D)

According to Euler's Theory

$$-\frac{M}{I} = \frac{\sigma}{y}$$

$$\Rightarrow \; M = -\frac{\sigma I}{c} = -\sigma \frac{bh^2}{6}$$

$$= -\frac{1200 \times 10^6 \times 40 \times 10^2 \times 10^{-4}}{6}$$

$$= 800 \text{ Nm.}$$

4. (D)

According to Euler's Law ,

$$\Rightarrow \; R = -\frac{Ey}{\sigma}$$

$$= \frac{200 \times 5}{800} = 1.25 \text{ mm}$$

5. (B)

For a simply supported beam, loaded at the centre

$$\delta = \frac{5WL^3}{384EI}$$

$$\therefore \; \delta \propto \frac{1}{I}, \text{ where } I = \frac{bh^3}{12}$$

$$\therefore \; \delta \propto \frac{1}{b}$$

$\therefore$ When b is doubled, δ becomes half.

$$\therefore \; \delta_2 = \delta_1 / 2$$

6. (A)

For (A),

R_A, R_B are the reactions.

Doing moment balance about support B,

$$R_A \times 1 + 100 = 1000 \times \frac{1}{4}$$

$$\Rightarrow \; R_A = +250 - 100 = 150 \text{ N} \uparrow$$

7. (A)

Doing a moment balance about point B,

$$R_A \times 2 + 50 = \int_0^1 100(1+x)\,dx$$

$$= 100 + \frac{100 \cdot 1^2}{2} = 150$$

$\Rightarrow$ $R_A = 50\text{ N} \uparrow$

8. (C)

Doing a moment balance about point B,

$$R_A \times 4 = \int_0^1 100(x+3)\,dx = \int_0^1 50x \cdot x\,dx$$

$$= 300 + \frac{100 \cdot 1^2}{2} + \frac{150 \cdot 1^3}{3}$$

$$= 300 + 50 + 50 = 400\text{ N}$$

$\Rightarrow$ $R_A = 100\text{ N}$

9. (B)

First we find the reaction at A

Doing moment balance about B,

$$R_A \times 2 = \int_1^2 100(2-x)\,x\,dx$$

$$= \frac{2 \times 100}{2}[4-1] - \frac{100}{3}[8-1]$$

$$= 300 - \frac{700}{3} = \frac{200}{3}$$

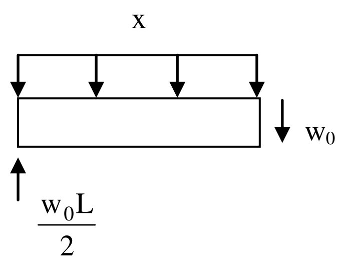
$$R_A = 33.33\text{ N}$$

$\therefore$ $V_a = 33.33 - \frac{1}{2} \times \frac{2}{3} \times \frac{2}{3} \times 100$

$$= \frac{100}{3} - \frac{200}{9} = \frac{100}{9}\text{ N}$$

10. (B)

Taking any section at distance x



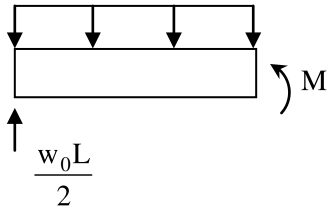
$$V + W_0 x = \frac{W_0 L}{2}$$

$$\Rightarrow V = \frac{W_0 L}{2} - W_0 x$$

which is equation of a straight line. $\therefore$ (B)

11. (C)

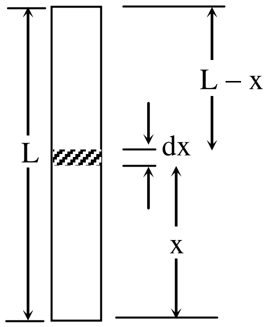
Taking a section at distance x,



$$\frac{W_0 L}{2}x - W_0 x \cdot \frac{x}{2} = M$$

$\therefore$ (C) is the answer.

12. (C)

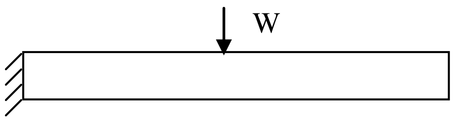


Taking an element of length dx at distance x from bottom the deflection δd is

$$\partial\delta = \frac{Pdx}{AE} \text{ where } P = wx$$

$$\therefore \quad \Delta = \int \partial\delta = \int_0^L \frac{wxdx}{AE} = \left.\frac{wx^2}{2AE}\right|_0^L = \frac{wL^2}{2AE}$$

13. (B)



Now, $\frac{\partial^2 y}{\partial x^2} = \frac{M}{EI}$

$$\frac{\partial y}{\partial x} = \frac{Mx}{EI} + C_1$$

Now $\left.\frac{\partial y}{\partial x}\right|_{x=0} = C_1 = 0$

$$\therefore \quad \frac{\partial y}{\partial x} = \frac{Mx}{EI}$$

Now, M at x = 0 is $\frac{WL}{2}$

$$\therefore \quad \frac{\partial y}{\partial x} = \frac{WL^2}{4EI}$$

14(a). (A)

14(b). (A)

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ANSWER KEY TO PRACTICE PROBLEMS

1.	(B)	2.	(D)	3.	(D)
4.	(D)	5.	(C)	6.	(C)
7.	(D)	8.	(D)	9.	(C)
10.	(A)	11.	(B)	12.	(A)
13.	(D)	14.	(A)	15.	(D)
16.	(D)	17.	(D)	18.	(D)
19.	(C)	20.	(B)	21.	(A)
22.	(D)	23.	(C)	24.	(D)
25.	(D)	26.	(D)	27.	(C)
28.	(C)	29.	(C)	30.	(A)
31.	(D)	32.	(B)	33.	(C)
34.	(C)	35.	(A)	36.	(B)
37.	(C)	38.	(A)	39.	(C)
40.	(D)	41.	(C)	42.	(D)
43.	(D)	44.	(C)	45.	(C)
46.	(D)	47.	(C)	48.	(B)
49.	(C)	50.	(B)	51.	(D)
52.	(D)	53.	(C)	54.	(A)
55.	(D)	56.	(A)	57.	(A)
58.	(B)	59.	(D)	60.	(A)
61.	(D)	62.	(A)	63.	(C)
64.	(B)	65.	(C)	66.	(A)
67.	(A)	68.	(D)	69.	(A)
70.	(A)	71.	(D)		

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MODEL SOLUTIONS TO PRACTICE PROBLEMS

1. (B)

Now $y = \frac{q_0 x}{24EI}\left[L^3 - 2Lx^2 + x^3\right]$

Now, $M = \frac{q_0 Lx}{2} - \frac{q_0 x^2}{2}$

and $\frac{\partial^2 y}{\partial x^2} = \frac{M}{EI}$

$\Rightarrow \frac{\partial y}{\partial x} = \frac{q_0 Lx^2}{4} - \frac{q_0 x^3}{6} + C_1$

$y = \frac{q_0 Lx^3}{12} - \frac{q_0 x^4}{24} + C_1 x + C_2$

Now, y(0) = 0 at v(L) = 0

$\therefore\ y = \frac{-q_0 x\left(L^3 - 2Lx^2 + x^3\right)}{24EI}$

$\therefore\ y = -\frac{q_0 L}{4}\left[L^3 - \frac{L^3}{8} + \frac{L^3}{64}\right]\frac{1}{24EI}$

$\therefore$ By symmetry, y_{max} occurs at x = L/2

$\therefore\ y = \frac{5q_0 L^4}{384EI}$

2. (D)

Volumetric strain = $\left(\sigma_1 + \sigma_2 + \sigma_3\right)\frac{[1-2v]}{E}$

$= \frac{(40+60+20)(1-0.5)}{2\times10^5} = 3 \times 10^{-4}$

3. (D)

4. (D)

5. (C)

Now, $M = \frac{Wb}{L}x$, for $0 \le x \le \frac{L}{2}$

For a concentrated load, W, bending moment at any section x

$M = \frac{W}{2}.x$

$\therefore\ U = \int_0^{L/2} \frac{W^2}{4}.\frac{x^2}{2EI} = \frac{W^2}{8EI}.\int_0^{L/2} x^2 dx$

$= \frac{W^2}{8EI}.\frac{x^3}{3} = \frac{W^2}{8EI}.\frac{L^3}{8\times3}$

$\therefore\ 2U = \frac{2.W^2L^3}{8\times8\times3EI} = \frac{1}{96}.\frac{W^2L^3}{EI}$

6. (C)

7. (D)

8. (D)

9. (C)

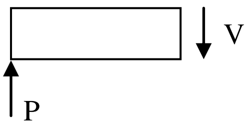
$$\frac{d^2M}{dx^2} = q \qquad \Rightarrow M = \int\left(\frac{w_0x^2}{2} + w_1x\right)dx$$

$$= \left.\frac{w_0x^3}{6} + \frac{w_1x^2}{2}\right|_{x=2} = \frac{4w_0}{3} + 2w_1$$

10. (A)

11. (B)

Taking section before 1m,



V = P
After 1m, v = 0
After 3 m, v = –p
Hence (B).

12. (A)

We know that

$$\frac{dM}{dx} = V$$

Now V = wns for 0 < x < 1, 3 < x < 4

∴ M is a straight line with slope P in this region and horizontal line in 1 < x < 3.

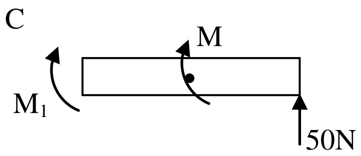
13. (D)

Now, taking moment balance at point A to find moment at support B

$R_B \times 4 = 100 + 100 \times 1 = 200$

$\Rightarrow \quad R_B = 50$ N

Now taking section C – C, and



doing moment balance

$50 \times 2 = 100 + M_1$

$\Rightarrow \quad M_1 = 0$

14. (A)

15. (D)

16. (D)

17. (D)

Deflection due to force $= \dfrac{WL^3}{3EI} = \dfrac{12 \times 2^3}{3EI} = \dfrac{32}{EI}$

Deflection due to moment M at free end

$$= \frac{ML^2}{2EI} = \frac{2M}{EI}$$

$\therefore \quad \frac{2M}{EI} = \frac{32}{EI}$

$\Rightarrow$ M = 16 k N–m

18. (D)

19. (C)

For a cantilever with force W,

$\delta_{max} = \frac{WL^3}{3EI}$

Also $M_{max} = WL$

$\therefore \quad \sigma_{max} = \frac{WLd}{2I} = \frac{M}{J/c} = \frac{Mc}{I} = \frac{W.L.d}{2I}$

$\therefore \quad \frac{\delta}{\sigma} = \frac{2L^2}{3Ed} = \frac{WL^3}{3EI} \times \frac{2}{W.Ld} = \frac{2}{3}.\frac{L^3}{Ed}$

20. (B)

21. (A)

22. (D)

23. (C)

24. (D)

25. (D)

26. (D)

27. (C)

Principal stresses

$= \frac{1}{2}\left(\sigma_x + \sigma_y\right) \pm \frac{1}{2}\sqrt{\left(\sigma_x - \sigma_y\right)^2 + 4\tau^2}$

$= \frac{1}{2}\left(100 + 20\right) \pm \frac{1}{2}\sqrt{(100 - 20)^2 + 4 \times 40^2}$

$= 60 \pm 56.57$

= 116.57 MPa, 3.43 MPa

28. (C)

For the principal plane,

$\tan 2\theta = \left(\frac{2\tau}{\sigma_y - \sigma_x}\right) = \frac{2 \times 20}{40} = 1$

$\Rightarrow \quad 2\theta = 45°$

$\Rightarrow \quad \theta = 22.5°$

29. (C)

$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{2.4 \times 500}{2 \times 5} = 120 \text{ MPa}$

30. (A)

$$\sigma_1 = \frac{pr}{2t} = 50\,\text{MPa}$$

Now, $P' = 2P$, $t' = t/2$, $r' = 4r$

$\therefore$ $\sigma' = 2 \times 2 \times 4 \times 50 = 800$ MPa

31. (D)

Stress = $\frac{pr}{t}$

$\Rightarrow$ $p = \frac{\sigma \times \tau}{r} = \frac{500 \times 2}{50} = 20$ kg/cm^2

32. (B)

From self wt. $\delta_1 = \frac{WL}{2AE}$

From load P, $\delta_2 = \frac{PL}{AE}$

$\therefore$ By superposition principle,

$$\delta = \delta_1 + \delta_2 = \frac{WL}{2AE} + \frac{PL}{AE}$$

33. (C)

34. (C)

Now, the point of 0 shear force occurs where the reaction on the left is balanced by the applied load

$\therefore$ First finding the reaction at support A,

$$R_A \times L = \frac{L}{2} \times KL \times \frac{L}{3} = \frac{KL^2}{6}$$

$$\Rightarrow R_A = \frac{KL^2}{6}$$

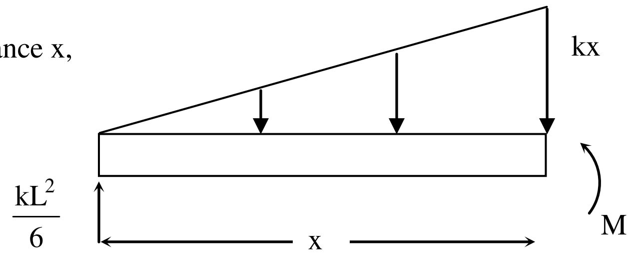
$\therefore$ Now, at any distance x,

$$\frac{KL^2}{6} = \frac{1}{2} \times Kx \times x$$

$$\Rightarrow x = \frac{L}{\sqrt{3}}$$

35. (A)

At any distance x,



$$M + K \times \frac{1}{2} \times x \times \frac{1}{3} x = \frac{KL^2}{6} \times x$$

$$M = \frac{Kx}{6}\left(L^2 - x^2\right)$$

$\therefore$ It is a parabola.

36. (B)

37. (C)

$$\delta_1 = \frac{WL^3}{2E\left(bh^3/12\right)}$$

$$\therefore \quad \delta_2 = \frac{W}{2}\cdot\frac{L^3}{2E\left(\frac{b(2h)^3}{12}\right)}$$

$$\therefore \quad \frac{\delta_2}{\delta_1} = \frac{1}{16}$$

38. (A)

Now, strain energy = $\int_0^L \frac{M^2}{2EI}dx$

For a point loading $M = \frac{P}{2}x$

$$\therefore \quad W_1 = 2\int_0^{L/2}\frac{P^2}{8EI}x^2dx = \frac{L^3}{96EI}$$

39. (C)

Maximum stress = $\frac{100}{0.8} = 125\text{MPa}$

40. (D)

Taking moment balance about point B,

$R_A \times 4 = \int_2^4 100\,x\,dx = \frac{100}{2}[16-4] = 600$

R_A = 150 N

41. (C)

Doing vertical balance

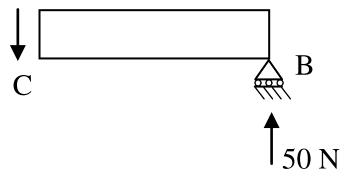
150 + R_B = 200 N

$\Rightarrow$ R_B = 50 N

42. (D)

To find shear force, we take the right body element,

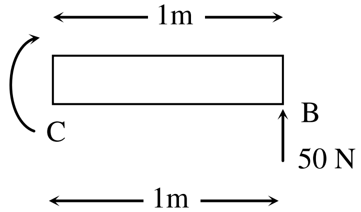
$\therefore$ V = 50 N



43. (D)

Taking moment about C,

M = 50 × 1 = 50 N – m

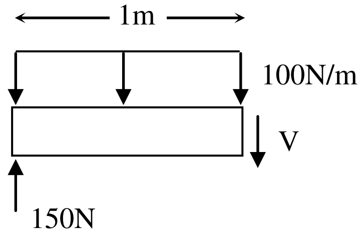


44. (C)

Doing vertical force balance,

150 = V + 100 × 1

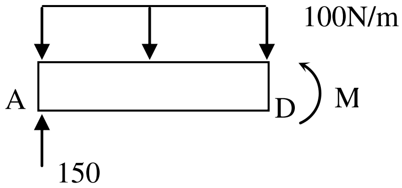
$\Rightarrow$ V = 50 N



45. (C)

Doing moment balance about point D

$150 \times 1 = M + 100 \times \frac{1}{2}$



(We write the distributed force as a single force acting at mid point)

$\Rightarrow$ M = 100 N – m

46. (D)

47. (C)

For cantilever beam,

δ at free end = $\frac{WL^3}{2EI}$

$\therefore \quad \delta \propto W$

Hence $\delta_1 / \delta_2 = 1/2$

48. (B)

Now, $\delta_1 = \frac{WL^3}{2EI}$

$$\delta_2 = \frac{W}{2} \cdot \frac{(2L)^3}{2EI} = \frac{4WL^3}{2EI}$$

$$\therefore \quad \frac{\delta_2}{\delta_1} = 4$$

49. (C)

50. (B)

51. (D)

52. (D)

$$\delta = \frac{5WL^3}{348EI}$$

$$\Rightarrow \quad \delta \propto \frac{1}{I}, \quad \text{where } I = \frac{bh^3}{12}$$

$$\Rightarrow \quad \delta \propto \frac{1}{h^3} \qquad \therefore \ \delta_2 = \frac{\delta_1}{8}$$

53. (C)

Now, for simply supported beam,

$$\delta \propto \frac{1}{EI} \quad \text{for same loading}$$

$\therefore$ As δ is also same,

$$E \propto \frac{1}{I}$$

$$\Rightarrow \frac{E_1}{E_2} = \frac{I_2}{I_1} = \frac{b_2 h_2^{\,3}}{b_1 h_1^{\,3}} = \frac{1}{8}$$

54. (A)
Now for a simply supported beam,

$$\delta_{max} = \frac{WL^3}{48EI}$$

and $M_{max} = \frac{WL}{4}$; $\quad Z = \frac{bd^2}{6} = \frac{2I}{d}$

Max bending stress = $\frac{M}{Z} = \frac{WLd}{8I}$

$$\therefore \frac{\delta}{\sigma} = \frac{L^2}{6ED}$$

55. (D)

56. (A)

57. (A)

58. (B)

59. (D)
Now, T is proportional to (N/H). And J/R contains the term R^3 which is proportional to R. Hence, R is proportional to D.

60. (A)

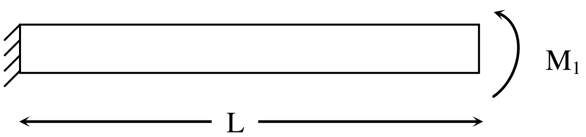
Now, T = $\frac{350}{0.3} \times \frac{\pi}{32} \times (0.6)^4$ = 14.84 kgcm

Power = $0.1484 \times 9.81 \times 2\pi \times \frac{10000}{60}$ = 1524 W.

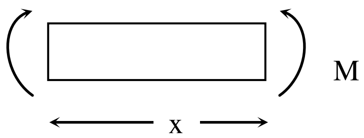
61. (D)
$W = \pi^2 EI/n\ell^2$, hence we see that it is related to the effective length ℓ.

62. (A)

63. (C)



The bending moment at any section C – C is



$M = M_1$

Now, $\frac{\partial^2 y}{\partial^2 x} = \frac{M}{EI}$

$\Rightarrow \frac{\partial y}{\partial x} = \frac{M_1 x}{EI} + C_1$

Now, $\frac{\partial y}{\partial x}(x = 0) = 0$

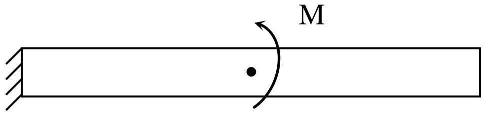
$\therefore C_1 = 0$

$$y = \frac{M_1 x^2}{2EI} + C_2$$

$$y = \left.\frac{M_1 x^2}{2EI}\right|_{x=L/2} \Rightarrow y = \frac{M_1 L^2}{8EI}$$

64. (B)

Now from 0 < x < 1/2



$$\frac{\partial^2 y}{\partial x^2} = \frac{M}{EI}$$

$$\Rightarrow \frac{\partial y}{\partial x} = \frac{M}{EI} \cdot x + C_1$$

Now, $C_1 =$ 0 as $\left.\frac{\partial y}{\partial x}\right|_{x=0} = 0$

$$\therefore \left.\frac{\partial y}{\partial x}\right|_{L/2} = \frac{ML}{2EI}$$

Also, $y = \frac{Mx^2}{2EI} + C_2,\ \ C_2 = 0$ as $y = 0|_{x=0}$

$\therefore\ \ y = \frac{ML^2}{8EI}$ at x = L/2

$\therefore$ Taking the equation of the line from x = L/2 till x = L,

y = mx + C,

$m = \frac{ML}{2EI}$

and $\frac{ML^2}{8EI} = \frac{ML}{2EI} \cdot \frac{1}{2} + C$

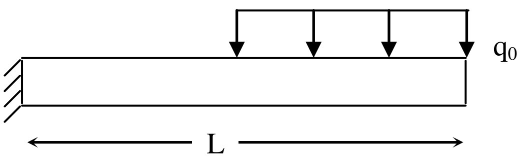
$$\Rightarrow C = -\frac{ML^2}{8EI}$$

$$\therefore\ \ y = \frac{MLx}{2EI} - \frac{ML^2}{8EI}$$

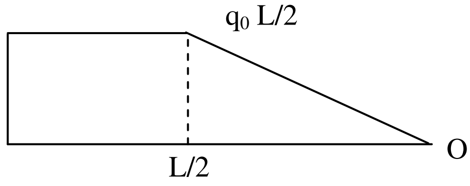
$\therefore$ at x = L

$$y = \frac{3ML^2}{8EI}$$

65. (C)



The shear force diagram for this looks like,



Now V = shear force $= EI\dfrac{\partial^3 y}{\partial x^3}$

$\therefore \quad \dfrac{\partial^2 y}{\partial x^2} = \dfrac{Vx}{EI} + C_1$ (Integrating the above once)

Now, $\quad \dfrac{\partial^2 y}{\partial x^2} = 0$ at $x = L$

$\therefore \quad C_1 = -VL / EI$

$$\frac{\partial y}{\partial x} = \frac{Vx^2}{2EI} - \frac{Lx}{EI} + C_2$$

Now, $\quad \dfrac{\partial y}{\partial x} = 0$ at $x = 0 \qquad \therefore C_2 = 0$

$$y = \frac{Vx^3}{6EI} - \frac{VLx^2}{2EI} + C_3$$

Again y = 0 at x = 0 $\qquad \therefore C_3 = 0$

$$\therefore \quad y = \frac{Vx^3}{6EI} - \frac{VLx^2}{2EI}$$

At $x = \dfrac{L}{2}$, $\qquad y = \dfrac{Vx^3}{48EI} - \dfrac{VL^3}{8EI} = -\dfrac{5VL^3}{48EI}$

$$\therefore \quad y = -\frac{5q_0L^4}{96EI}$$

66. (A)

Now, $\quad M = \dfrac{q_0Lx}{2} - \dfrac{q_0x^2}{2}$

$$\therefore \quad \frac{\partial^2 v}{\partial x^2} = \frac{M}{EI}$$

$$\Rightarrow \quad \frac{\partial y}{\partial x} = \frac{q_0Lx^2}{4} - \frac{q_0x^3}{6} + C_1$$

$$y = \frac{q_0Lx^3}{12} - \frac{q_0x^4}{24} + C_1x + C_2$$

Now, y(0) = 0 at y(L) = 0

$$\therefore \quad y = \frac{-q_0x\left(L^3 - 2Lx^2 + x^3\right)}{24EI}$$

$$\therefore \quad y = -\frac{q_0L}{4}\left[L^3 - \frac{L^3}{8} + \frac{L^3}{64}\right]\frac{1}{24EI}$$

$$= -\frac{q_0L^4}{96EI}\left[\frac{64-8+1}{64}\right] = \frac{57q_0L^4}{6144EI}$$

67. (A)

Hoop stress is given by = pr/t
Longitudinal stress is given by = pr/2t
Hence, their ratio is 1/2

68. (D)

T = 119 × hp/f = 119 × 15 / 20 = 89.25 N.m

Now, $\frac{J}{R} = \frac{T}{\tau_m}$ = 1487.5 mm^3

$I_p/c = \pi c^3/2$. Therefore, c = 9.82 mm

Diameter is around 20 mm

69. (A)

Now, stress = α t E

$= 12\times10^{-6}\times20\times200 = 48 MPa (T)$

70. (A)

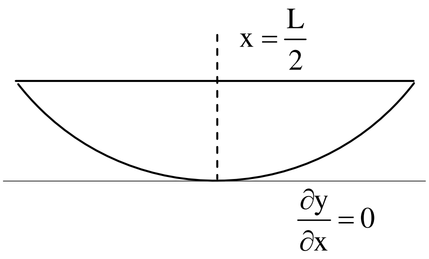
Now Stress = α ΔT E

$= 2 \times 10^{-6} \times 10^{6} \times 50 = 100\ kg/cm^2$

As the temperature is decreased, the body contracts and the stress acting are tensile.

71. (D)

From the diagram and symmetry



We can see that at L/2, $\frac{\partial y}{\partial x} = slope = 0$

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