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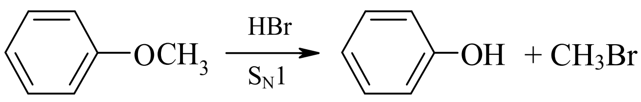
IIT JEE 2010 : Solution to Paper – I

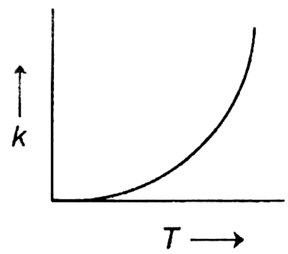
CHEMISTRY

1. (D)



2. (A)

$k = Ae^{-E_a/RT}$



3. (B)

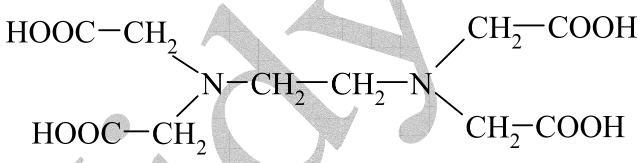
The enthalpy of Cl_2 in its standard state is zero.

4. (C)

Bond energy of C−C = 346 kJ/mol ≈ 100 kcal/mol.

5. (C)

EDTA = ethylenediamine-tetraacetic acid



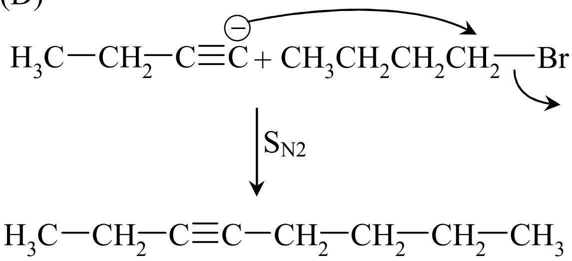
They have six donor atoms.

6. (B)

Ionisation Isomerism arises when the co-ordination compounds give different ions in solution.

Correct option is compound −B i.e., $[Cr(H_2O)_4Cl_2](NO_2)$.

7. (D)



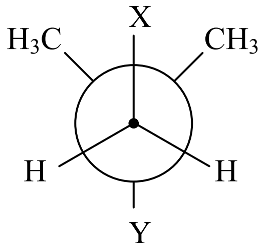
(3−octyne)

8. (A)
Pyranose = six member Ring, i.e., Ring (a)
Furanose = five member Ring, i.e., Ring (b)
Ring (a) have α-Glycosidic linkage and Ring-(b) have β-Fructosidic linkage.

9. (C), (D)

HNO_3 and CH_3COOH	→ Mixture of strong acid and weak acid doesn't form buffer.
KOH and CH_3COONa	→ No buffer.
HNO_3 and CH_3COONa	→ $\underset{\text{salt}}{CH_3COONa} \rightarrow \underset{\text{base}}{CH_3COO^-}$
CH_3COOH and CH_3COONa	→ Weak acid and its salt forms acidic buffer.

10. (B), (D)



→ X and Y both cannot be H and H. Option (A) is false.

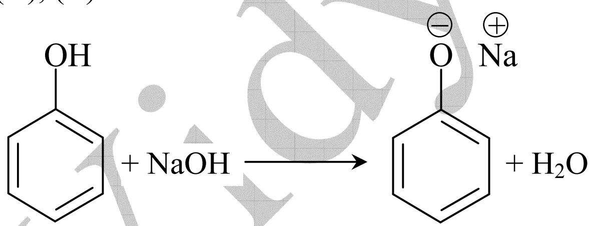
→ H and C_2H_5 as X and Y represents 2, 2-dimethyl butane, hence option (B) is correct.

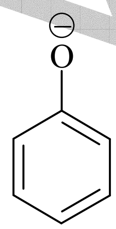
→ C_2H_5 and H as X and Y does not represent 2,2-dimethyl butane

→ CH_3 and CH_3 as X and Y represents 2,2-dimethyl butane hence option (D) is correct.

11. (A), (B)
Resistance and Heat capacity depends upon the amount of matter present.

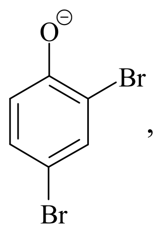
12. (A), (C)

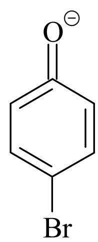




Phenoxide

is strong ortho and para directing.

Possible intermediate are 

, 

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13. (B), (C), (D)

Temporary hardness is due to the presence of bicarborates of Calcium and Magnesium.

→ Temporary hardness can be removed by Clark's process which involves the addition of slacked lime [$Ca(OH)_2$].

$$Ca(HCO_3)_2 + Ca(OH)_2 \rightarrow 2CaCO_3 \downarrow + 2\,H_2O$$

→ Washing Soda removes the both the temporary and permanent hardness.

$$CaCl_2 + Na_2CO_3 \rightarrow CaCO_3 + 2NaCl$$

$$Ca(HCO_3)_2 + Na_2CO_3 \rightarrow CaCO_3 \downarrow + 2\,NaHCO_3$$

→ By passing chlorinated water.

Solution 14 to 16

Memory map for the Extraction of Silver

Silver ore
(Argentite)

↓

CONCENTRATION BY FROTH FLOATATION PROCESS

Powdered ore + Water + Pine oil + Air ⟶ Froth carrying sulphide particles

↓

CYANIDATION

Concentrated or + NaCN solution (0.4 – 0.6%) + Air

$$Ag_2S + 4\,NaCN \rightleftharpoons 2\,NaAg\,(CN)_2 + Na_2S$$

$$4\,Na_2S + 5\,O_2 + 2\,H_2O \longrightarrow 2\,Na_2SO_4 + 4\,NaOH + 2\,S$$

Filtration

↓

PRECIPITATION OF SILVER WITH ZINC

$$2NaAg(CN)_2 + Zn \longrightarrow 2Ag + Na_2Zn(CN)_4$$

Black ppt.

$$\text{Black ppt. of Ag} + KNO_3 \xrightarrow{\text{Fuse}} \text{Compact mass (Silver metal)}$$

↓

ELECTROLYTIC REFINING

Anode : Impure silver
Cathode : Pure silver plate
Electrolyte : $AgNO_3$ solution + HNO_3
Pure silver deposited on the cathode

14. (B)

15. (D)

$FeSiO_3$ (as Slag)

16. (C)

$$2Cu_2O + Cu_2S \rightarrow 6Cu + SO_2$$

(S: −2 → +4)

17. (B)

M(s) | M^+ (aq; 0.05 molar) || M^+ (aq; 1 molar) | M(s)

i.e., Concentration cell.

Oxidation half cell : $M(s) \rightarrow M^+$ (aq; 0.05 molar) + e^-

Reduction half cell : M^+ (aq; 1 molar) + $e^- \rightarrow M(s)$

Overall cell reaction : M^+ (1 molar) $\rightarrow M^+$ (0.05 molar)

$$E = 0 - \frac{0.0591}{1}\log_{10}\left(\frac{0.05}{1}\right)$$

$$= +0.076 \text{ V}$$

i.e.,$E_{Cell} > 0$ and $\Delta G < 0$.

18. (C)

$$E = -\frac{0.0591}{1}\log_{10}\left(\frac{0.0025}{1}\right)$$

$$= 0.153 \text{ V}$$

$$= 140 \text{ mV}$$

19. [3]

(Beryl) $Be_3Al_2Si_6O_{18}$

Cyclic or ring silicates have general formula (SiO_3^{2-}) or $(SiO_3)_n^{2n-}$.

20. [3]

25.2 mL	$\rightarrow$	3 significant figure
25.25 mL	$\rightarrow$	4 significant figure
25.0 mL	$\rightarrow$	3 significant figure

Average value = 25.15 mL should also have 3 significant figure (minimum).

21. [0]

$$\text{rate} = K\left(\frac{\text{Initial concentration} - \text{final concentration}}{(t_2 - t_1)}\right)$$

$$\frac{d(c)}{dt} = \text{constant} = 5$$

$$= \frac{1-0.75}{0.05} = \frac{1-0.4}{0.12}$$

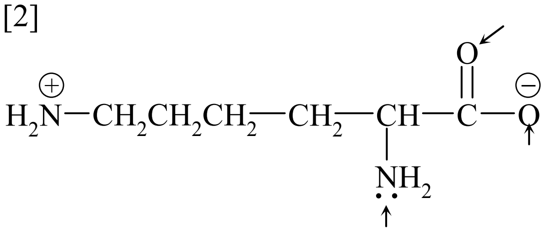
$$= \frac{1-0.1}{0.18}$$

i.e., zeroth order reaction.

22. [3]

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{142}_{54}Xe + ^{90}_{38}Sr + 4\,^{1}_{0}n$$

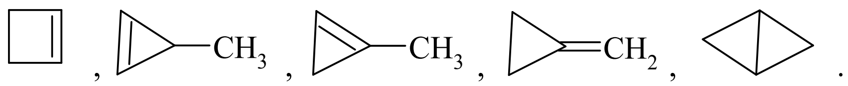
23. [2]



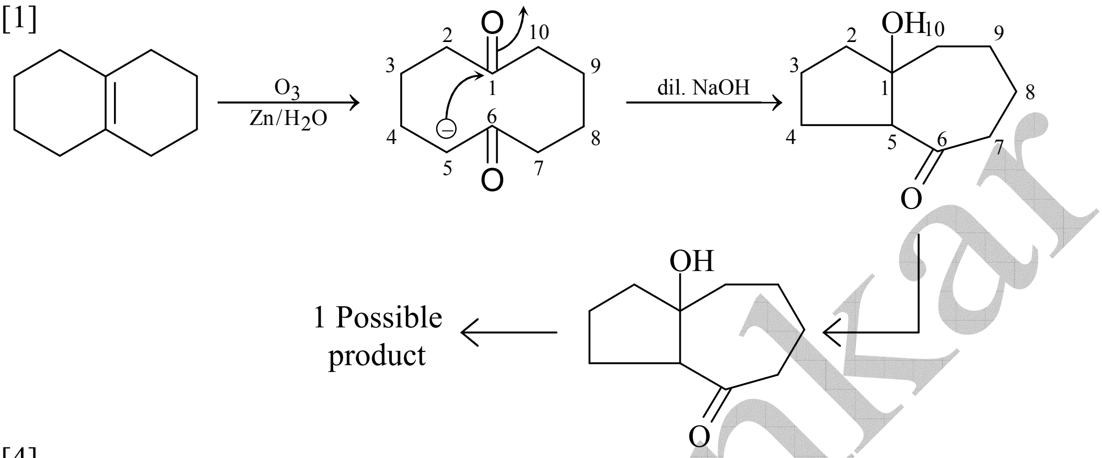
24. [5]

DBE of C_4H_6 $= \dfrac{C_4H_{10} - C_4H_6}{2} = \dfrac{4}{2}$

$= 2$



25. [1]



26. [4]

Acidic compounds are soluble in aq. NaOH, alcohols are less acidic than water.

27. [3]

Basic compound :

KCN is salt of strong base and weak acid → Basic

K_2SO_4 is salt of strong base and strong acid → Neutral

$(NH_4)_2C_2O_4$ is salt of weak acid and weak base → Acidic

$K_b = 1.8 \times 10^{-5}$, Ka of $H_2C_2O_4$

$pK_b = 4.74$ $\quad pK_1 = 1.2$

$$pH = 7 + \frac{1}{2}(pK_a - pK_b) = 7 + \frac{1}{2}(1.2 - 4.74)$$

pH < 7 i.e. acidic.

NaCl → Neutral

$Zn(NO_3)_2$ → Acidic

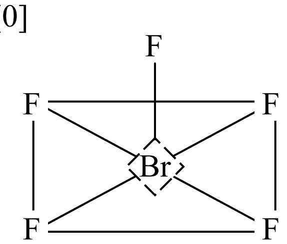
$FeCl_3$ → Acidic

K_2CO_3 → Basic

NH_4NO_3 → Acidic

LiCN → Basic

28. [0]



All four planar bonds (F−Br−F) will reduce from 90° to 85° due to lone pair – bond pair repulsion.

MATHEMATICS

29. (A)

Linear system of equations posses either unique solution or infinite number of solutions. Hence no matrix A is possible.

30. (B)

Using L' Hospital's rule,

$$\ell = \lim_{x\to 0} \frac{x\log(1+x)}{(x^4+4)3x^2}$$

$$= \lim_{x\to 0} \frac{\log(1+x)}{x} \times \frac{1}{3(x^4+4)} = \frac{1}{12}$$

31. (B)

$$\alpha + \beta = -p$$

$$\alpha^3 + \beta^3 = q \quad \Rightarrow \quad (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = q$$

$$\Rightarrow \; -p^3 - 3\alpha\beta(-p) = q$$

$$\Rightarrow \; \alpha\beta = \frac{p^3+q}{3p}$$

Now, required sum, $S = \dfrac{\alpha^2+\beta^2}{\alpha\beta} = \dfrac{p^2 - \dfrac{2(p^3+q)}{3p}}{\dfrac{p^3+q}{3p}}$

$$= \left(\frac{p^3-2q}{p^3+q}\right)$$

Product, p = 1

$\therefore$, (B) is correct.

32. (D)

$$f'(x) = 2x\left[e^{x^2} - \frac{1}{e^{x^2}}\right] \geq 0 \text{ for all } x \in (0,1).$$

Similarly, g(x) and h(x) are also increasing functions.

$\therefore$ a = f(1), b = g(1), c = h(1)

$$\Rightarrow a = b = c = e + \frac{1}{e}.$$

33. (D)

$$2B = A + C$$

$$\Rightarrow 2B = \pi - B$$

$$\Rightarrow B = \pi/3$$

Now, $\dfrac{a}{c}\sin 2C + \dfrac{c}{a}\sin 2A$

$$\Rightarrow \frac{\sin A}{\sin C} \times 2\sin C\cos C + \frac{\sin C}{\sin A} \times 2\sin A\cos A$$

$$= 2\sin(A+C) = 2\sin B = \sqrt{3}$$

34. (C)

If $ax + by + cz + d = 0$ is equation of plane containing the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$, then

$d = 0$ and $2a + 3b + 4c = 0$ …(1)

Also, direction of normal of a plane containing the straight lines

$\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is

$$\begin{vmatrix} i & j & k \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix} = 8\,\hat{i}-\hat{j}-10\hat{k}$$

Now, since these two planes are perpendicular,

thus $8a - b - 10c = 0$ …(2)

From (1) and (2), $\frac{a}{1}=\frac{b}{-2}=\frac{c}{1}$

$\therefore$, (C) is correct.

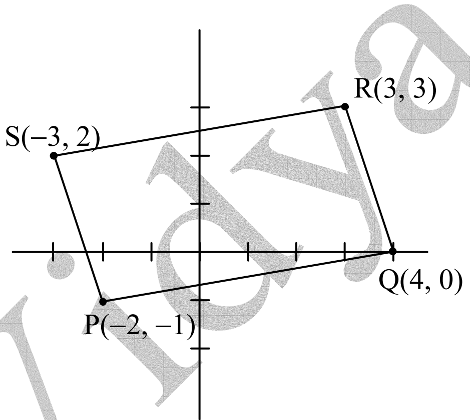
35. (C)

If $\omega^{r_1}+\omega^{r_2}+\omega^{r_3}=0$, then r_1, r_2, r_3 are of the type 3n, 3n + 1, 3n + 2 ($n \in N$) in any order .

In a die, there are two numbers of each type.

$\therefore$, Required probability = $\frac{3!\times 2\times 2\times 2}{6\times 6\times 6}=\frac{2}{9}$

36. (A)



From the graph, it is clear that (A) option is correct.

37. (A)

Put $x = \tan\theta$

$$\int_0^{\pi/4}\frac{\tan^4\theta(1-\tan\theta)^4\sec^2\theta\,d\theta}{(1+\tan^2\theta)}$$

$$= \int_0^{\pi/4}\tan^2\theta(1-\tan\theta)^4\left(\sec^2\theta-1\right)d\theta$$

$$= \int_0^{\pi/4}\tan^2\theta(1-\tan\theta)^4\sec^2\theta\,d\theta-\int_0^{\pi/4}\tan^2\theta(1-\tan\theta)^4\,d\theta$$

Put, $\tan\theta = t$ and Solve further.

38. (A), (C), (D)

$Z = (1 - t)\, z_1 + t\, z_2$

If $t \in (0, 1)$, then z_1, z, z_2 are collinear with z lying between z_1 and z_2.

$\therefore$ (A) and (D) are correct

Now $\quad z - z_1 \;=\; t(z_2 - z_1)$

$$\therefore \begin{vmatrix} z - z_1 & \overline{z - z_1} \\ z_2 - z_1 & \overline{z_2 - z_1} \end{vmatrix} = \begin{vmatrix} t\,(z - z_1) & t\,\overline{(z_2 - z_1)} \\ (z_2 - z_1) & \overline{\overline{(z_2 - z_1)}} \end{vmatrix}$$

$$= (z_2 - z_1)\,\overline{(z_2 - z_1)} \begin{vmatrix} t & t \\ 1 & 1 \end{vmatrix} = 0$$

$\therefore$ (C) is also correct.

39. (B), (C)

40. (B)

$\angle ACB = \dfrac{\pi}{6}$

$$\therefore \cos C = \frac{\sqrt{3}}{2} = \frac{\left(x^2 + x + 1\right)^2 + \left(x^2 - 1\right)^2 - (2x+1)^2}{2\left(x^2 + x + 1\right)\left(x^2 - 1\right)}$$

$\Rightarrow \sqrt{3}\left(x^2 + x + 1\right) = 2x^2 + 2x - 1$

$\Rightarrow x = -(2 + \sqrt{3}),\ 1 + \sqrt{3}$

$\Rightarrow x = 1 + \sqrt{3} \quad$ [$\because$ sum of two sides of a Δ is greater than the 3rd side]

41. (C), (D)

Let $A \equiv \left(t_1^2, 2t_1\right)$ and $B \equiv \left(t_2^2, 2t_2\right)$

$$\therefore \text{Slope of diameter} = \frac{2\left(t_1 - t_2\right)}{\left(t_1^2 - t_2^2\right)} = \frac{2}{t_1 + t_2}$$

$\because$ Circle touches x –axis,

y – coordinate of centre = $|\,r\,|$

$$\Rightarrow \frac{2\left(t_1 + t_2\right)}{2} = |\,r\,|$$

$$\therefore \text{Slope} = \frac{2}{|\,r\,|} = \pm\frac{2}{r}$$

42. (D)

$A = \begin{bmatrix} a & b \\ c & a \end{bmatrix}$ a, b, c $\in$ {0, 1, 2, .., P–1}

$|A| = a^2 - bc$

If $a^2 - bc$ is divisible by 'P' means

$a^2 = bc \mod P.$

For symmetric b = c

So, $a^2 = b^2 \mod P$

$\therefore\ \ a^2 - b^2$ is divisible P

$\Rightarrow$ (a − b) (a + b) is divisible by P $\Rightarrow$ (a + b) is divisible P.

So, no. of such cases is P − 1

And no. of matrices whose determinant is O is P.

So, $P + (P - 1) = 2P - 1$

43. (C)

44. (D)

45. (B)

Write the given options in the form of y = mx + c and substitute the values of m and c in the condition of tangency; i.e. $c = \pm r\sqrt{1+m^2}$ and $c = \pm\sqrt{a^2m^2 - b^2}$

46. (A)

Solving the two curves simultaneously, we get point of intersection $\left(6, \pm 2\sqrt{2}\right)$

$\therefore$ Equation of circle is $x^2 + y^2 - 12x + 24 = 0$

47. (1)

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$$

$$c_1^1 \rightarrow c_1 + c_2 + c_3$$

$$\left(z + 1 + \omega + \omega^2\right)\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & z+\omega^2 & 1 \\ 1 & 1 & z+\omega \end{vmatrix} = 0$$

$$R_2^1 \rightarrow R_2 - R_1$$
$$R_3^1 \rightarrow R_3 - R_2$$

$$z\begin{vmatrix} 1 & \omega & \omega^2 \\ 0 & z+\omega^2-\omega & 1-\omega^2 \\ 0 & 1-\omega & z+\omega-\omega^2 \end{vmatrix} = 0$$

$$z.\left[\left\{z + (\omega^2 - \omega)\right\}\left\{z - (\omega^2 - \omega)\right\} - \left\{(1-\omega)\,(1-\omega)^2\right\}\right] = 0$$

$$\Rightarrow z.\left[z^2 - \left(\omega^4 + \omega^2 - 2\omega^3\right) - \left(1 - \omega - \omega^2 + 1\right)\right] = 0$$

$$\Rightarrow z.\left[z^2 - \left(\omega + \omega^2 - 2\right) - (3)\right] = 0$$

$$\Rightarrow z\left[z^2 + 3 - 3\right] = 0$$

$$\Rightarrow z^3 = 0$$

$$z = 0$$

48. (2)

$$S_k = \frac{\frac{(k-1)}{k!}}{1-\frac{1}{k}} = \frac{1}{(k-1)!}$$

$$\therefore \frac{100^2}{100!} + \sum_{k=1}^{100}\left(\frac{k^2-3k+1}{(k-1)!}\right)$$

$$= \frac{100}{99!} + \sum_{k=1}^{100}\frac{(k-1)(k-2)-1}{(k-1)!}$$

$$= \frac{100}{99!} + \sum_{k=1}^{100}\left(\frac{1}{(k-3)!} - \frac{1}{(k-1)!}\right)$$

$$= \frac{100}{99!} + \frac{1}{0!} - \frac{1}{2!} + \frac{1}{1!} - \frac{1}{3!} + \ldots + \frac{1}{97!} - \frac{1}{99!}$$

$$= \frac{100}{99!} + 2 - \frac{1}{98!} - \frac{1}{99!}$$

$$= 2$$

49. [3]

50. [9]

Equation of tangent is $\frac{Y-y}{X-x} = \frac{dy}{dx}$

Putting, $X = 0$

We get, $Y = y - x\frac{dy}{dx}$

$$\therefore\ y - x\frac{dy}{dx} = x^3$$

$$\Rightarrow x\frac{dy}{dx} - y = -x^3$$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{-1}{x}\right)y = -x^2$$

On solving this linear D.E., we get,

$$y = -\frac{x^3}{2} + \frac{3x}{2}$$

$$\therefore\ f(-3) = 9$$

51. [3]

$$\tan\theta = \cot 5\theta$$

$$\Rightarrow \tan\theta = \tan\left(\frac{\pi}{2} - 5\theta\right)$$

$$\Rightarrow 6\theta = n\pi + \frac{\pi}{2}$$

$$\Rightarrow \theta = \frac{(2n+1)\pi}{12}$$

$\Rightarrow \theta = \frac{-5\pi}{12}, \frac{-3\pi}{12}, \frac{-\pi}{12}, \frac{\pi}{12}, \frac{3\pi}{12}, \frac{5\pi}{12} \quad \left(\because, \theta \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)\right)$

$\cos 4\theta = \sin 2\theta$

$\cos(4\theta) = \cos\left[\frac{\pi}{2} - 2\theta\right]$

$\Rightarrow 4\theta = 2n\pi \pm \left(\frac{\pi}{2} - 2\theta\right)$

$\Rightarrow \theta = \frac{(4n+1)\pi}{12}$ or $\frac{(4n-1)\pi}{4}$

$\Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{-3\pi}{12}$

$\therefore$ 3 common solutions.

52. [2]

Maximum value of expression $\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$

$\Rightarrow$ Minimum value of expression $\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta$

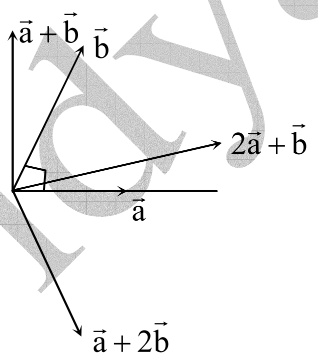
$= 1 + 4\cos^2\theta + \frac{3}{2}\sin 2\theta = 1 + 2 + 2\cos 2\theta + \frac{3}{2}\sin 2\theta$

$= 3 - \sqrt{4 + \frac{9}{4}} = \frac{1}{2}$

$\therefore$ Maximum value of the required expression = 2

53. [5]

$\vec{a}$ and $\vec{b}$ are unit vector perpendicular each other.



$2\vec{a}+\vec{b}$ and $\vec{a}-2\vec{b}$ lying in the plane of $\vec{a}$ and $\vec{b}$ and also perpendicular each other.

$\vec{a}\times\vec{b}$ unit vector and $\vec{a}\times\vec{b}\times(\vec{a}-2\vec{b})$ is a vector parallel to $(2\vec{a}+\vec{b})$

As can be seen from fig.

$\therefore$ Value is $\left|2\vec{a}+\vec{b}\right| \times \left|\vec{a}-2\vec{b}\right|$

$= \sqrt{5} \times \sqrt{5} = 5$

54. (2)

The line 2x + y = 1 passes through $\left(\frac{a}{e}, 0\right)$.

$\therefore 2\frac{a}{e} = 1 \quad \Rightarrow \quad a = \frac{e}{2}$

Also by condition of tangency

$4a^2 - b^2 = 1$

$\Rightarrow 4.\frac{e^2}{4} - b^2 = 1$

$\Rightarrow e^2 = 1 + b^2$

$\Rightarrow e^2 = 1 + a^2e^2 - a^2$

$\Rightarrow e^2 = 1 + \frac{e^4}{4} - \frac{e^2}{4}$

$\Rightarrow 4e^2 = 4 + e^4 - e^2$

$\Rightarrow e^4 - 5e^2 + 4 = 0$

$e^2 = 4$ or $e^2 = 1$

$e = 2$

55. (6)

Equation of plane containing the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is

$a(x-1) + b(y-2) + c(z-3) = 0$ …(1)

and $2a + 3b + 4c = 0$ …(2)

Also, plane (1) & $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are $\perp^r$

$\Rightarrow 3a + 4b + 5c = 0$ …(3)

From (2) & (3)

$\frac{a}{-1} = \frac{b}{2} = \frac{c}{-1}$

so, put in (1) equation of plane is

$x - 2y + z = 0$

so, distance between two planes is $= \left|\frac{d-0}{\sqrt{1+4+1}}\right| = \sqrt{6}$

$\Rightarrow |d| = 6$

56. (4)

f (x) { x } if [x] is odd

1 − { x } if [x] is even

Graph of f (x) is symmetric along y–axis. Hence f (x) is an even function.

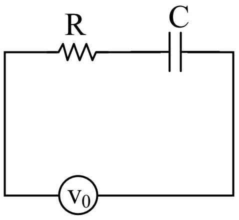
$\therefore \frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos(\pi x)\,dx = \frac{\pi^2}{5}\int_0^{10} f(x)\cos(\pi x)\,dx = 5\times\frac{\pi^2}{5}\int_0^2 f(x)\cos(\pi x)\,dx$

$= \pi^2\left[\int_0^1 (1-x)\cos\pi x + \int_1^2 (x-1)\cos(\pi x)\,dx\right]$

$= 4$ (put (1 − x) = t)

PHYSICS

57. (B)



$$\text{Impedance} = \sqrt{\left[R^2 + X_C^{\,2}\right]}$$

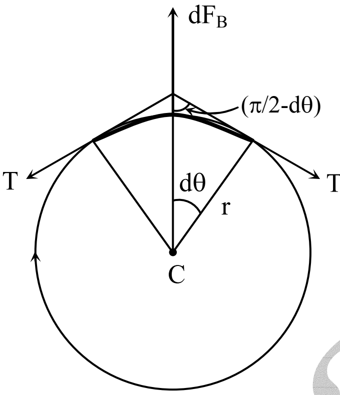
$$Z = \sqrt{\left[R^2 + \frac{1}{\omega^2 C^2}\right]}$$

If ω is increased; Impedance will decrease

$$\text{current amplitude } I = \frac{\xi_{in}}{Z} = \frac{V_0}{\sqrt{\left[R^2 + \frac{1}{\omega^2 C^2}\right]}}$$

with increasing ω; Z decreases and hence I increases.
Thus the bulb will glow brighter.

58. (C)



Consider an element of length dL subtending 2dθ at centre.

$$2T\ \cos\left(\frac{\pi}{2} - d\theta\right) = dF_B = I(dL)B$$

$$2T\ \sin\,(d\theta) = I\ r\ 2\ d\theta\ .\ B$$

$$\Rightarrow\ T = \frac{IrB}{(\sin d\theta)/d\theta} = \frac{IBL}{2\pi\left(\frac{\sin d\theta}{d\theta}\right)}$$

In the limit $d\theta \to 0$

$$T = \frac{IBL}{2\pi}$$

59. (A)

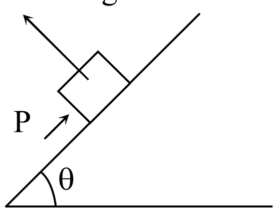
$P_1 = mg(\sin\theta - \mu\cos\theta)$

If net force is to be zero;

$m(g\sin\theta - \mu\cos\theta) + f - mg\sin\theta = 0$

$\Rightarrow\ f_1 = \mu mg\cos\theta$ (up the incline)

for P_2 ; $f_2 = -\mu mg\cos\theta$ (down the incline)



60. (A)

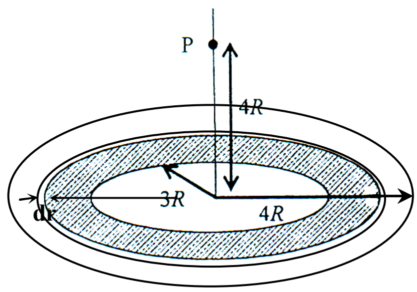
$$\text{Mass per unit area} = \frac{M}{\pi(4R)^2 - \pi(3R)^2} = \frac{M}{7\pi R^2}$$

$$\text{Mass of elemental ring} = (2\pi r\ .\ dr)\ \frac{M}{7\pi R^2}$$

$$\therefore\ dm = \frac{2M}{7R^2} r.dr$$

Potential at P due to dm

$$= -\frac{Gdm}{\sqrt{r^2 + 16R^2}} = -\frac{2GM}{7R^2}\frac{r.dr}{\sqrt{r^2 + 16R^2}}$$



Potential due to whole disc = $-\dfrac{GM}{7R^2}\displaystyle\int_{3R}^{4R}\dfrac{2r.dr}{\sqrt{r^2+16R^2}}$

$t = r^2 + 16R^2$

$\Rightarrow$ $dt = 2r\,dr$

$\therefore$ $V = -\dfrac{GM}{7R^2}\displaystyle\int_{25R^2}^{32R^2}\dfrac{dt}{\sqrt{t}} = -\dfrac{GM}{7R^2}\dfrac{\sqrt{t}}{(1/2)}\Bigg|_{25R^2}^{32R^2}$

$\therefore$ $V = -\dfrac{2GM}{7R^2}\left(4\sqrt{2}\,R - 5R\right) = -\dfrac{2GM}{7R}\left(4\sqrt{2}-5\right)$

Work done in taking unit mass from P to ∞ = 0 − (V) = $\dfrac{2GM}{7R}\left(4\sqrt{2}-5\right)$

61. (C)

It is known;

$R = \dfrac{\rho\ell}{A}$

For the given square sheet

$\ell = L$

$A = L\,t$

$R = \dfrac{\rho L}{Lt}$ $\Rightarrow$ $R = \dfrac{\rho}{t}$.

Hence (C).

62. (D)

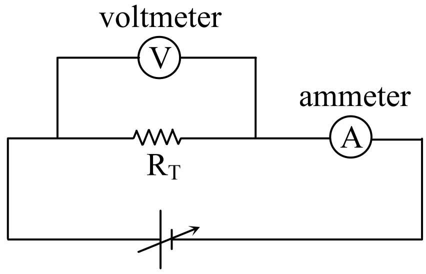
63. (D)

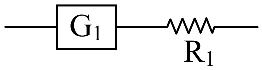
$P = \dfrac{V^2}{R}$

$\therefore$ $P_{100} > P_{60} > P_{40}$ $\Rightarrow$ $\dfrac{1}{R_{100}} > \dfrac{1}{R_{60}} > \dfrac{1}{R_{40}}$

64. (C)

To verify Ohm's law, the student must be able to measure the voltage across R_T and current through R_T.

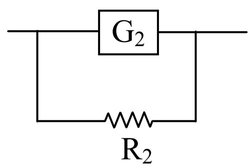




Voltmeter is connected in parallel.

$\Rightarrow$ Its resistance must be high.

$\Rightarrow$ R_1 is connected in series.



Ammeter is connected in series.

$\Rightarrow$ Its resistance must be low.

$\Rightarrow$ R_2 is connected in parallel.

65. (A), (B), (C), (D)

Internal energy depends only on temperature.

$T_A = T_B \quad \therefore\ U_A = U_B.$

AB is isothermal process :

$$\therefore W = nRT_0 \ell n\left(\frac{V_2}{V_1}\right)$$

$$= P_0 V_0 \ \ell n\left(\frac{4V_0}{V_0}\right) = P_0V_0 \ \ell n\, 4.$$

For B – C ; $\frac{V}{T}$ is constant

$$\therefore \frac{4V_0}{T_0} = \frac{V_0}{T_c} \Rightarrow T_c = \frac{T_0}{4}$$

For CA; V is constant

$$\therefore \frac{P_c}{T_c} = \frac{P_A}{T_A} \Rightarrow P_A = P_0,\ T_A = T_0$$

$$P_c = T_c \times \frac{P_0}{T_0} = \frac{P_0}{4}.$$

66. (A), (C)

$m_1 = 1\text{ Kg} \qquad V_1 = 2\text{ms}^{-1}$

$m_2 = 5\text{Kg} \qquad V_2 = ?$

$0 \rightarrow \qquad \leftarrow 0 \qquad 0 \rightarrow$

$m_1u_0 \qquad m_1v_1 \qquad m_2v_2$

$$V_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u$$

$$\Rightarrow -2 = \left(\frac{1-5}{1+5}\right)u \quad \Rightarrow \quad u = \frac{2\times 6}{4} = 3\text{ ms}^{-1}$$

$$V_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u \quad \Rightarrow \quad \frac{2}{6}\times 3 = 1\text{ms}^{-1}$$

Total momentum of system = $m_1u_0 = 1\text{kg}\times 3\text{ms}^{-1} = 3\text{kgms}^{-1}$

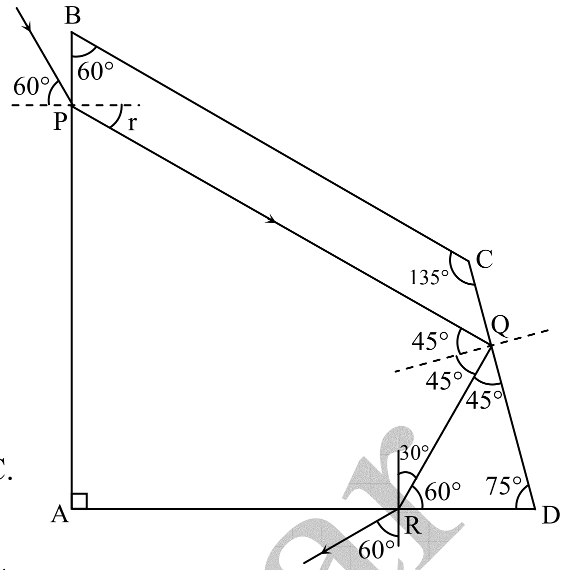
Momentum of 5 kg body after collision = 5 × 1 = 5 kgm^{-1}

Total K.E. = $\frac{1}{2}m_0{u_0}^2 = \frac{1}{2}\times 9 = \frac{9}{2}$ J

$$\text{K.E. of C.M.} = \frac{1}{2}(m_1 + m_2)\left[\frac{m_1u_0}{m_1 + m_2}\right]^2 = \frac{{m_1}^2{u_0}^2}{2(m_1 + m_2)}$$

$$= \frac{1\times 9}{2\times 6} = \frac{9}{12} = 0.75\ \text{J}.$$

67. (A), (B)



$$\frac{\sin 60}{\sin r} = \sqrt{3} \quad \Rightarrow \sin r = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{1}{2}$$

$\Rightarrow \quad r \quad = 30°$

Ray PQ || BC

$\angle PQC = 360° - (60 + 90 + 30 + 135)$

$\quad = 180 - 135° = 45°$

$\sin \theta_c = \frac{1}{\sqrt{3}}$

$\therefore \quad \theta_c < 45°$

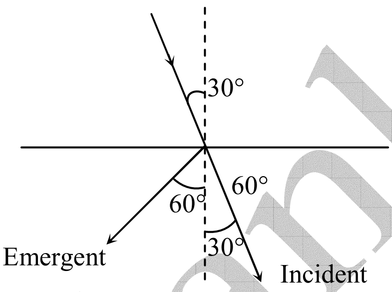
The ray suffers total internal reflection at face DC.

Angle of incidence at AD = 30°

$$\frac{\sin 30°}{\sin r'} = \frac{1}{\sqrt{3}}$$

$\Rightarrow \sin r' = \sqrt{3} \times \sin 30° = \frac{\sqrt{3}}{2}$

$\therefore \quad r' = 60°$



Clearly the angle between emergent and incident rays is 90°

68. (A), (D)

Time period ; $T = \frac{t}{m}$ (t = time for m oscillations)

$$\frac{\Delta T}{T} = \frac{\Delta t}{t}$$

$\Rightarrow \quad \Delta T = \left(\frac{\Delta t}{t}\right) T$

$\Delta t = 1$ second (ℓ.c. of stopwatch)

$t = 40$ and $T = 2$ seconds.

$\therefore \quad \Delta T = \frac{2 \times 1}{40} = 0.05$ sec

We know; $T = 2\pi \sqrt{\frac{L}{g}} \Rightarrow g = \frac{4\pi^2 \ell}{T^2}$

$$\therefore \quad \frac{\Delta g}{g} = \frac{2\Delta T}{T} = 2 \times \frac{1}{40} = \frac{1}{20} = 5\%$$

69. (A), (D)

Density of Electric field lines $\propto$ strength of Electric field.

$\therefore$ Q_1 is more in magnitude

$|Q_1| > |Q_2|$

Electric field will be zero at a finite distance to the right of Q_2.

70. (C)
Periodic motion can occur if particle loses all its kinetic energy by the time it reaches X_0.
$\therefore \quad E < V_0$.

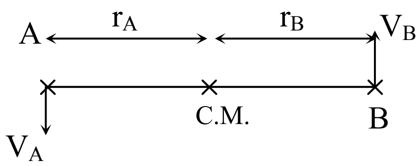
71. (B)
Using dimesional analysis;
$[\alpha] = ML^2T^{-2}$ x $L^{-4} = ML^{-2}\ T^{-2}$
$$\left[\frac{1}{A}\sqrt{\frac{m}{\alpha}}\right] = \left[L^{-1}\sqrt{\left[\frac{M}{ML^{-2}T^{-2}}\right]}\right] = T$$

72. (C)
For $|x| > x_0$
$V = V_0$
$\therefore \quad F = 0 \qquad (x > x_0)$

73. (A)
$B_2 > B_1 \Rightarrow T_c(B_2) < T_c(B_1)$
Only (A) satisfies.

74. (B)
$T_c(B)$ decreases with B.
$T_c(7.5) < T_c(5) < T_c(0)$
$75K < T_c(5) < 100\,K$
$T_c(10) < 75\,K$

75. [6]
Let the origin of co-ordinates be at C.M.
$m_A r_A = m_B r_B \qquad \ldots\ldots(1)$



The required ratio is $\dfrac{m_A V_A r_A + m_B V_B r_B}{m_B V_B r_B}$

using (1) $\quad \dfrac{V_A + V_B}{V_B} = 1 + \dfrac{V_A}{V_B}$

The necessary centripetal force is provided by the force of gravitational attraction.
$$\frac{m_A {V_A}^2}{r_A} = \frac{m_B {V_B}^2}{r_B}$$
$$\frac{{V_A}^2}{{V_B}^2} = \frac{m_B}{m_A} \times \frac{r_A}{r_B}$$
Using (1),
$$\frac{{V_A}^2}{{V_B}^2} = \frac{{m_B}^2}{{m_A}^2} \Rightarrow \frac{V_A}{V_B} = \frac{m_B}{m_A} = \frac{11M_S}{2.2\,M_S}$$
$$= \frac{1}{0.2} = 5$$
$\therefore$ The required ratio is 6.

76. [3]

$g = \frac{GM}{R^2}$

$M = \rho \times \frac{4}{3}\pi R^3$, where ρ is the mass density.

$g \propto (\rho \times R)$

$\frac{g_{planet}}{g_{earth}} = \frac{\rho_P \times R_P}{\rho_E \times R_E}$

$\frac{\sqrt{6}}{11} = \frac{2}{3} \times \frac{R_P}{R_E}$

$\therefore \quad R_P = \left(\frac{3\sqrt{6}}{22}\right) R_E$

Escape velocity = $V = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2GM}{R^2} \times R} = \sqrt{g \times R}$

$$\frac{V_P}{V_E} = \sqrt{\frac{g_P}{g_E} \times \frac{R_P}{R_E}} = \sqrt{\frac{\sqrt{6}}{11} \times \frac{3\sqrt{6}}{22}} = \sqrt{\frac{3 \times 6}{(11)^2 \times 2}}$$

$$\frac{V_P}{V_E} = \sqrt{\frac{3^2}{(11)^2}} = \frac{3}{11}$$

$\therefore \quad V_P = \frac{3}{11} \times 11 = 3\,\text{km/s}$.

77. [8]

Heat absorbed = Heat absorbed ($-5°C \to 0°C$) + Heat absorbed for melting 1 gm of ice.

= mS ΔT + 1 gm × L

$420 = m \times 10^{-3} \times 2100 \times 5 + 1 \times 10^{-3} \times 3.36 \times 10^5$

420 = m × 10.5 + 336

84 = m × 10.5

$m = \frac{840}{105} = 8$ gm.

78. [7]

Let the velocities of the cars be v_1 and v_2

frequency detected by car 1 = $f_0\left(\frac{v_0 + v_1}{v_0}\right) = f'$

v_0 = speed of sound in air

For echo, car be comes the source of frequency f'

$\therefore$ frequency of echo received = $f'\left(\frac{v_0}{v_0 - v_1}\right)$

$$f_1 = f_0\left(\frac{v_0 + v_1}{v_0 - v_1}\right)$$

Similarly, $f_2 = f_0\left(\frac{v_0+v_2}{v_0-v_2}\right)$

$$\frac{f_1-f_2}{f_0} = \left(\frac{v_0+v_1}{v_0-v_1}\right) - \left(\frac{v_0+v_2}{v_0-v_2}\right) = \left(\frac{1+\frac{v_1}{v_0}}{1-\frac{v_1}{v_0}}\right) - \left(\frac{1+\frac{v_2}{v_0}}{1-\frac{v_2}{v_0}}\right)$$

$$= \left(1+\frac{v_1}{v_0}\right)\left(1-\frac{v_1}{v_0}\right)^{-1} - \left(1+\frac{v_2}{v_0}\right)\left(1-\frac{v_2}{v_0}\right)^{-1}$$

$$= \left(1+\frac{v_1}{v_0}\right)\left(1+\frac{v_1}{v_0}\right) - \left(1+\frac{v_2}{v_0}\right)\left(1+\frac{v_2}{v_0}\right) \quad \left(\because \frac{v_1}{v_0} \; \& \; \frac{v_2}{v_0} \text{ are small}\right)$$

$$\therefore \quad \left(\frac{f_1-f_2}{f_0}\right) = \left(1+\frac{v_1}{v_0}\right)^2 - \left(1+\frac{v_2}{v_0}\right)^2$$

$$\therefore \quad \frac{2(v_1-v_2)}{v_0} = \frac{f_1-f_2}{f_0}$$

$$\Rightarrow \quad v_1 - v_2 = \frac{v_0}{2}\left(\frac{f_1-f_2}{f_0}\right) = \frac{330\text{ms}^{-1}}{2}\times 0.012$$

$$= 330 \times 0.006 = 33 \times 0.06 = 1.98 \text{ m/s}$$

In kmph, $v_1 - v_2 = \frac{18}{5}\times 1.98 = 7$ kmph.

79. [6]

$$m = \frac{f}{f+u}$$

$$\frac{m_{25}}{m_{50}} = \frac{f+u_{50}}{f+u_{25}} = \frac{20-50}{20-25} = \frac{-30}{-5} = 6$$

80. [3]

Energy gained E = qV, where V is the potential difference.

$$\frac{E_\alpha}{E_p} = \frac{q_\alpha}{q_p} = 2$$

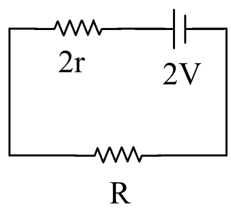
$E = \frac{p^2}{2m}$ where p is the linear momentum.

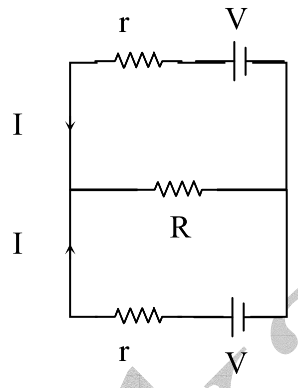
$$\therefore \quad \frac{\frac{P_\alpha^2}{2m_\alpha}}{\frac{P_p^2}{2m_p}} = 2 \quad \Rightarrow \quad \left(\frac{P_\alpha}{P_p}\right)^2 \times \frac{m_P}{m_\alpha} = 2$$

$$P = \frac{h}{\lambda} \quad \Rightarrow \quad \left(\frac{\lambda_p}{\lambda_\alpha}\right)^2 \times \frac{1}{4} = 2$$

$$\frac{\lambda_p}{\lambda_\alpha} = 2\sqrt{2} = 2 \times 1.41 = 2.82 \sim 3.$$

81. [4]





$$J_1 = \left(\frac{2V}{R+2r}\right)^2 \times R = \frac{4V^2}{(R+2r)^2} \times R$$

$$J_2 = (2I)^2 \times R = 4 \times I^2 R$$

Consider the lower loop:

$$V - Ir - 2I \times R = 0$$

$$I = \frac{V}{r+2R}$$

$$J_2 = \frac{4V^2}{(r+2R)} \times R$$

$$\frac{J_1}{J_2} = 2.25 = \frac{(r+2R)^2}{(R+2r)^2}$$

$$1.5 = \frac{r+2R}{R+2r}$$

$$1.5 = \frac{1+2R}{R+2} \quad \Rightarrow \quad 1.5R + 3 = 1 + 2R$$

$$2 = 0.5R \quad \Rightarrow \quad R = 4\Omega$$

82. [9]

$$\frac{R_A}{R_B} = \frac{e\sigma A_A T_A{}^4}{e\sigma A_B T_B{}^4} = \frac{r_A^2}{r_B^2} \times \frac{T_A{}^4}{T_B{}^4}$$

$$= \left(\frac{6}{18}\right)^2 \times \left(\frac{\lambda_B}{\lambda_A}\right)^4 = \frac{1}{3^2} \times \left(\frac{1500}{500}\right)^4 = \frac{1}{3^2} \times 3^4 = 3^2 = 9$$

83. [5]

$$y = 4\sin(2x - 6t) + 3\sin\left(2x - 6t - \frac{\pi}{2}\right) = 4\sin(2x-6t) - 3\cos(2x-6t)$$

$$= 5 \times \left[\frac{4}{5}\sin(2x-6t) - \frac{3}{5}\cos(2x-6t)\right] = 5 \times \sin(2x - 6t - \theta)$$

$\therefore$ Amplitude = 5.

84. [4]

$$Y = \frac{\text{stress}}{\text{strain}} \Rightarrow \text{stress} = Y_{\text{strain}}$$

$$\Rightarrow \quad \frac{F}{A} = \frac{Y\,\Delta y}{L} \quad \Rightarrow \quad F = \left(\frac{Y\Delta y}{L}\right)A \qquad \text{[Restoring]}$$

$$\therefore \quad a = \left(\frac{YA}{Lm}\right)\Delta y$$

$$\omega^2 = \frac{YA}{Lm} \Rightarrow Y = \frac{\omega^2 \times Lm}{A}$$

$$\Rightarrow Y = \frac{140 \times 140 \times 1 \times 0.1}{4.9 \times 10^{-7}} = 400 \times 10^7\ Nm^2$$

$\therefore$ Y = 4×10^9 Nm^{-2}.

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